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CONSUMER SURPLUS DEMAND FUNCTIONS AND POLICY ANALYSIS

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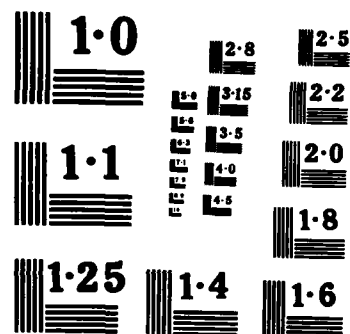
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Consumer Surplus, Demand Functions, and Policy Analysis

Frank Camm

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Consumer Surplus, Demand Functions, and Policy Analysis

Frank Camm

June 1983

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PREFACE

Policy analysts familiar with the concept of consumer surplus typically think of it as the area under a demand curve. Unfortunately, except in cases most suitable for textbooks, consumer surplus can rarely be represented by simple areas under demand curves. In particular, when redistributive government policies affect consumer surplus, attempts to measure such effects with areas under demand curves are likely to lead to severe errors.

This report explains how demand functions can be used properly to measure policy-induced changes in consumer surplus. For the most part, it brings together existing results from the economics literature and presents them in a common, systematic framework. Its goal is to provide the practicing policy analyst with a rigorous and intuitive understanding of the most common measures of consumer surplus used today. The text is written for a reader with a solid introductory understanding of microeconomics--either upper-division undergraduate courses or graduate training in a professional school--and a practical familiarity with elementary calculus. No prior understanding of consumer surplus, welfare measurement, or cost-benefit analysis is assumed.

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SUMMARY

Consumer surplus is a monetary measure of the difference between what an individual pays for consuming a good or service and the amount he is willing to pay, given his income and the prices he faces. It is basically the net monetary benefit he receives by consuming the good. Policies that change his income or the price he faces can change the amount of net benefit he receives from consumption and his monetary valuation of that benefit. Hence, they change his consumer surplus. Measures of changes in consumer surplus basically translate policy effects into monetary terms that we can use to determine (a) how policy changes affect groups of individuals in terms of a commonly understood and accepted unit of measure and (b) ultimately whether such changes are worthwhile from a social point of view.

THREE MEASURES OF CONSUMER SURPLUS

Many measures of consumer surplus are possible. The three most commonly used are Hicksian, Marshallian, and Harberger measures. Hicksian measures, named for John Hicks, ask one of two simple questions: First, given the effects of a policy, what change in an individual's money income will maintain his well-being constant? For example, what increase in money income would prevent a price increase from injuring the individual? The change in money income is the Hicksian compensating variation. Second, given the effects of a policy, what change in an individual's money income will affect him the same way the proposed policy change would if it occurred? For our example, what reduction in money income would hurt the individual exactly as much as the price increase would? This change in money income is the Hicksian equivalent variation.

Marshallian measures, named for Alfred Marshall, ask a different question. They hold money income constant but recognize that changes in money income affect an individual's sense of well-being. A Marshallian measure uses the rate at which well-being or "utility" changes with money income as a conversion factor to turn around and translate changes

in utility into monetary terms. For example, when money income remains constant, a price increase hurts an individual by reducing his utility. A Marshallian measure seeks a monetary value for that reduction in utility.

Harberger measures, named for Arnold Harberger, take an approach that combines the two measures above. In terms of our example, he asks first why the price rose. If it rose in response to a tax increase, for example, he notes that the tax revenue collected must go somewhere; he makes a case for assuming that it ultimately returns to the individual as a lump sum. That is, in the spirit of a Hicksian measure, Harberger compensates the individual with an increase in money income for a *portion* of the injury imposed by a price increase. But he recognizes in general that full compensation is impossible in a closed economy. A tax-induced price increase must lead to some drop in utility, despite the compensation offered. Harberger then uses a Marshallian approach to measure the monetary value of the tax-induced drop in utility.

These measures differ because the different forms of compensation considered in each case affect the demand for goods as policies change. By effectively holding utility constant, Hicksian measures assume that compensation is available so that demand is defined by income-compensated demand functions as policies change. By holding money income constant, Marshallian measures assure that no compensation occurs; demand is defined by uncompensated demand functions. Harberger measures assume partial compensation; they define demand along the general equilibrium or "Bailey" demand functions that define the extent of compensation available. As an empirical matter, these various demand functions are very similar unless (a) the good in question--for example, housing--accounts for a large share of an individual's expenditures or (b) the individual's use of the good changes markedly as his income changes--that is, the extent of compensation markedly affects his consumption of the good. When these functions do differ, the three measures above of changes in consumer surplus will also differ. We must be sure to understand what these differences mean for the policy analyst and the policymaker.

CONSUMER SURPLUS AND DEMAND CURVES

The first thing the analyst must be aware of is how each measure relates to demand functions. The association above between the types of compensation underlying measures of consumer surplus and different types of demand *functions* often raises the expectation that we can associate each of these measures with the demand *curves* that these functions generate. Textbook presentations of consumer surplus promote this expectation by defining consumer surplus as the area under a demand curve. This is appropriate only if (a) the good in question has no close substitutes or complements or (b) consumer prices for all other goods are fixed. Such circumstances, of course, are typical of the situations we expect to find in textbooks; they rarely occur in the real world.

For practical purposes, consumer surplus is not the area under a demand curve: It is the sum of areas to the left of consumption loci. We can see this by understanding that a change in consumer surplus is the difference between the change in an individual's willingness to pay for goods and the change in the amount he must pay for them. Willingness to pay for a good is simply the price reflected in the individual's demand function for that good at just the point where consumption occurs. For small changes in consumption, Δx_i , then, the change in willingness to pay for the i th good is $p_i \Delta x_i$, where p_i is the demand price of x_i . For small changes in consumption, the change in the amount an individual must pay, $\Delta(p_i x_i)$, is approximately $x_i \Delta p_i + p_i \Delta x_i$. The difference between these changes--the change in consumer surplus--is simply $-x_i \Delta p_i$, the area of a narrow sliver just to the left of any point of consumption. Summing over groups of such slivers yields an area to the left of a consumption locus for the i th good. Summing areas for all goods whose prices are affected by a policy yields a measure of the change in consumer surplus.

These loci differ from demand curves for two principal reasons. First, demand curves are defined by holding all prices but one constant. But if a good has substitutes or complements, a change in its price will change the demands for the substitutes and complements. When this occurs, their demand curves shift and, unless supply curves for the

substitutes/complements are flat, their prices change. Such price changes shift the demand curve in the market where price first changed. As a result, we cannot use the demand curve in a market to trace out changes in consumption as prices in that market change. And because consumer surplus must be tied to consumption, we cannot tie it to the demand curve.

Second, when a price change in one market affects consumption elsewhere, as in the case just considered, consumers beyond the market where prices first changed are affected. As consumption changes in these "secondary" markets, prices also change unless supply is perfectly elastic. These changing prices affect consumer surplus. As above, the change can be represented by a series of narrow slivers with area $-x_i \Delta p_i$. A group of such slivers, defined by the path of consumption in the secondary market, defines an effect on consumer surplus that must be considered. This path of consumption does not follow a single demand curve, though each point on it corresponds to one point on some demand curve. Again, because consumer surplus must be tied to consumption, we cannot tie it in any meaningful way to a demand curve in a secondary market.

In the end, then, each measure of consumer surplus is associated with one type of demand function. Given a demand function, a policy change will trace out consumption loci in each market affected by the policy change. Each measure must be tied to areas associated with these loci, not with the demand curves in the markets. Differences in the measures are linked to differences in these loci.

POINTS TO CONSIDER IN APPLICATION

First, does the analyst or decisionmaker believe that the existing distribution of income is ideal? All measures of consumer surplus are based on an individual's willingness to pay for goods, *given* his income and the prices he faces. If the distribution of income is not ideal, we must give more weight to the effects of policy on individuals with "too little" income than on those with "too much." How much more weight depends on how far we are from the ideal. Political decisionmakers are unlikely all to agree that *any* distribution of income is ideal, no matter which distribution we choose. This suggests that the analyst

should conduct disaggregated analysis of consumer surplus. He should aggregate individuals into groups that decisionmakers are likely to believe have common interests and leave to the decisionmaker the problem of how to weight policy effects on these different groups.

Second, does the analyst or decisionmaker want an actual or a hypothetical monetary measure of a policy's effects on individuals? If he wants an actual measure, he must use the measure based on the level of compensation actually envisioned. If no compensation is envisioned, a Marshallian measure is appropriate. If some compensation is envisioned, either as a direct cash payment or as an indirect provision of valuable services, a measure *like* the Harberger measure is appropriate. It should be scaled, however, to the level and type of compensation actually expected, not that embodied in a Bailey demand curve. If we intend exactly to preserve the current well-being of a group of individuals, a Hicksian compensating variation measure is most appropriate. If the decisionmaker does not want an actual measure, we must ask ourselves why. The most compelling reason raises the third point to consider in applying consumer surplus measures to a policy problem.

Third, does the analyst or decisionmaker want a well-behaved welfare indicator with which to rank alternative policies in social terms? If so, only Hicksian measures offer such an indicator. All other measurers display the following difficulty: If a policy or policy package affects several prices, these measures depend on the order in which we adjust prices from their prepolicy to their postpolicy levels. They can display different values for a given set of price changes depending on how we get from one set of prices to the next. This means that consumer surplus cannot be expressed as a well-defined function of prices and hence cannot function as a well-behaved welfare indicator. Hicksian measures alone avoid this problem and one in particular, the Hicksian equivalent variation, has especially attractive characteristics as a welfare indicator. This does not mean measures other than Hicksian measures are necessarily poorly defined or take arbitrary values; each yields a unique value for the change in consumer surplus along any specific path of price changes. When "path-independent" measures are considered desirable, however, only Hicksian measures qualify.

In most instances, Hicksian, Marshallian, and Harberger measures are hard to distinguish empirically. Because Marshallian measures are typically the easiest to obtain--they depend solely on observable, uncompensated demand functions--they are probably the best to use when differences are not expected. When differences are likely to be important, most economists will prefer a Hicksian measure. It is the only measure that produces a well-defined welfare indicator that economists can use to rank alternatives in social terms. Policy analysts and decisionmakers are unlikely to believe that any one ranking is clearly dominant. Rankings are likely to differ among decisionmakers with different views of who deserves the greatest attention in public policy. For them, disaggregated measures of policy effects based on the actual compensation scheme contemplated are likely to be most useful. Such measures will be similar to Harberger measures. Greater attention to the effects of in-kind and cash compensation on demand behavior will be required before such measures can be perfected.

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I. INTRODUCTION

Consumer surplus is a monetary measure of the difference between what an individual pays for consuming a good or service and the amount he is willing to pay, given his income and the prices he faces. It is basically the net monetary benefit or "rent" he receives by consuming the good. Policies that change his income or the prices he faces can change the amount of net benefit he receives from consumption and his monetary valuation of that benefit. Hence, they change his consumer surplus. Measures of changes in consumer surplus basically translate policy effects into monetary terms that we can use to determine, first, how policy changes affect groups of individuals in terms of a commonly understood and accepted unit of measure, and ultimately, whether such changes are worthwhile from a social point of view.

The current theoretical literature on consumer surplus and social welfare measures seems to be reaching a consensus that well-defined ("path-independent") measures of changes in consumer surplus based on Hicksian income-compensated demand functions can be developed strictly as functions of observable data on quantities and prices. (See, for example, Hausman, 1981; McKenzie and Pearce, 1982.) This is a powerful result that could ultimately settle the theoretical debate among economists about how best to measure consumer surplus. Despite this result, other measures of consumer surplus are likely to remain in use for policy analysis. This is true because

- Hicksian measures of compensating and equivalent variation need not reflect exactly the welfare concept policy analysts want to measure.
- Non-Hicksian measures based on behavioral or Marshallian demand functions often show greater compatibility with measures of tax payments and receipts and other rents that are also relevant to social welfare and that change when policies change.

- Moving from observable data to exact Hicksian measures remains difficult enough that many policy analysts prefer non-Hicksian measures, if only as approximations of the Hicksian measures, when Hicksian and non-Hicksian measures have similar values.

This report reviews the relationship between several measures of consumer surplus and demand functions. It approaches this relationship with a particular point of view: Consumer surplus is an economist's device which, if it is to be useful in policymaking, must embody the concerns of the policymaker. That is, the monetary measure of personal or social well-being that a specific form of consumer surplus provides must make intuitive sense to the policymaker. And the analyst must apply this measure of consumer surplus in a way that facilitates decisionmaking. Because consumer surplus is first and foremost an *economist's* device, its precise definition and use have not often been examined from the point of view of the *policymaker*, perhaps its most important user. This report attempts to bridge the gap between the economist and the policymaker to assure that the concept of consumer surplus is used to best advantage.

It was tempting to orient this report heavily toward specific applications to help the policymaker understand what consumer surplus is. But quite frankly, we were more concerned that the policy-oriented economist know exactly what consumer surplus is to assure that he can facilitate its transfer into what may be unfamiliar and even inhospitable territory. Hence, this report starts with the price theoretic basics and builds the three most popular forms of consumer surplus in applied work today from first principles:

- The "Marshallian" measure, based on the uncompensated or behavioral demand function.
- The "Hicksian" measures--compensating and equivalent variation--based on the hypothetical Hicksian income-compensated demand function.

- The "Harberger" measure, based on the hypothetical general equilibrium or Bailey income-compensated demand function.

It shows how to transform these into the geometric analogs--"areas under demand curves"--that policy analysts typically use to calculate these measures and the exact relationships of these analogs to the underlying measures themselves. Despite fairly clear theoretical development, considerable confusion continues about how to translate the explicit mathematical expression that defines each measure of consumer surplus into the right set of areas associated with demand curves. This report shows that demand levels in the mathematical expressions all refer to *consumption loci* and that most confusion can be avoided by relying on these loci rather than on the demand curves underlying them.

And the report shows how to transform consumer surplus measures for individuals into measures relevant to social policy. Consumer surplus is least controversial when presented at the individual level. Moving beyond the individual is inherently a value-laden exercise that requires the policymaker's direct input. Any aggregate measure of social well-being based on consumer surplus must reflect the relevant policymaker's views about the relative importance of individuals or groups of individuals affected by a potential policy change. The report shows how to reflect these views in a rigorous way and suggests an approach for the analyst that minimizes his need to compare the worth of alternative groups, an inherently political activity best left to the policymaking arena.

A simple methodology is used throughout the report. First, the exact effects of infinitesimal changes in policy are derived for each type of consumer surplus. These effects are referred to throughout the report as *incremental* effects. Then a set of such incremental changes is chosen to represent a finite change in policy. The set of changes chosen is referred to throughout the report as a *path*, for reasons that become apparent in the text. A path may be chosen on the basis of the form that policy change is expected to take over time or more or less arbitrarily when a finite policy change occurs instantaneously. Finally, the set of incremental effects included in a path are summed to

yield a finite measure of change in consumer surplus. This finite measure is referred to throughout the report as a measure of *discrete* change in consumer surplus. The presentation of this methodology serves two purposes. It provides a rigorous basis for the results developed here. And it gives the reader a way to move beyond these results when he is faced with more complex policy changes than those addressed in the report.

For the most part, this report draws on separate discussions of measures of consumer surplus in the literature to bring together in one place a coherent discussion of

- the relationships among the three measures presented here,
- the problems associated with using estimates based on demand curves to quantify them,
- the problems associated with moving from individual to aggregate measures, and
- the factors a policy analyst might consider in choosing one measure over the other, implementing the one he chooses, or interpreting a policy analysis based on any one of these measures.

Section II develops the concept of consumer surplus at the level of the individual and contrasts the concepts associated with Hicks, Marshall, and Harberger. Section III explains how these concepts are related to different kinds of demand curves and to areas to the left of policy-induced consumption loci. Section IV discusses how to aggregate measures of consumer surplus across individuals and integrates the notion of consumer surplus with more general concerns about how policies affect social welfare. Section V closes the report with some notes on how to choose a measure of consumer surplus for a particular policy problem.

II. AN INDIVIDUAL'S CONSUMER SURPLUS

Policy changes affect an individual by affecting his income or the prices he pays for goods. That is, an individual's utility (U) is a function of the amounts of goods (x_i) he consumes,

$$U = U(x_1, \dots, x_n) \quad (2.1)$$

and the amounts of goods he consumes are functions of prices (p_i) and income (y):

$$x_i = x_i(p_1, \dots, p_n, y) \quad (2.2)$$

Hence, we study a policy's effect on individuals by tracing its effects through Eqs. (2.2) to (2.1). This section uses this approach to develop three different approaches to consumer surplus under a set of simplifying assumptions:

- We examine only the effects of price changes, although, as should become clear, we can easily use the approaches shown here to measure the effects of income changes as well.
- Producer prices are fixed and price changes for individuals result from taxes on or subsidies to the individuals.¹ Section IV lifts the assumption of fixed producer prices when the discussion moves beyond the individual to a broader view of applied welfare analysis.
- All goods are normal; individuals increase their demand for every good when money income rises. The concepts and methods described here can be used to examine the effects of price changes for inferior goods as well, but the discussion of such

¹Producer prices in a market are equal to consumer prices, less any per unit taxes in the market.

goods would only obscure the basic points this report aims to make.

With these points in mind, this section starts by introducing the key points to be made with a graphical two-good case. Then each point is discussed in more detail for n goods using, for the most part, elementary calculus.

GRAPHICAL EXPLANATION FOR TWO GOODS

Suppose an individual consumes only two goods, x_1 and x_2 , and a tax, t_1 , is imposed on x_1 . What can we say about how that effective price change, from p_1^a to $p_1^a + t_1 = p_1^b$, affected him? Figure 2.1 shows this case. The individual is endowed with a money income that allows him to consume any combination of x_1 and x_2 along Y^a before the price change; he maximizes utility within his budget by consuming at a . The

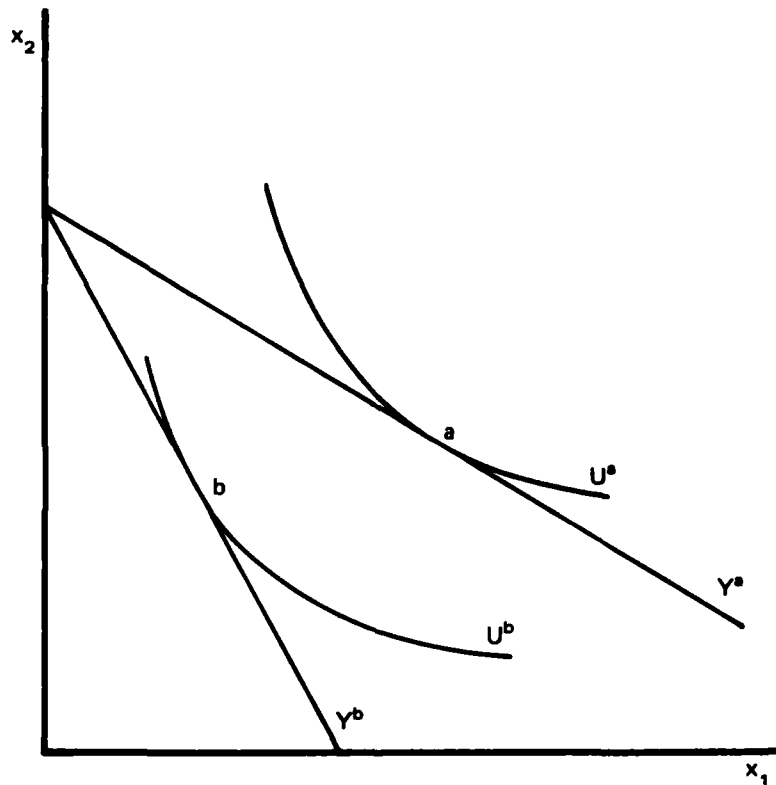


Fig. 2.1 — Change in consumption when one price rises

price change cuts his disposable income from y^a to $y^a - t_1 x_1$, thereby moving the budget line from Y^a to Y^b . The individual maximizes his utility within this new opportunity set at b. U^a and U^b show his levels of utility before and after the change. In terms of Eqs. (2.1) and (2.2), the policy change has changed the set of goods demanded in Eq. (2.2) (from a to b) and thereby has moved utility in Eq. (2.1) from U^a to U^b . How can we assign a monetary value to this change in utility? Three approaches are currently used.

The most widely used, the "Hicksian," asks how much the individual would have to be paid to accept the price change voluntarily. This can be approached in two ways. First, we can ask how much the individual's income would have to be raised following the price change to achieve the level of utility he enjoyed before the price change. Figure 2.2 illustrates how this is done.² Suppose we maintain the prices after the change and increase income until the individual can attain his initial level of utility. This effectively moves the individual's budget line until it becomes tangent to U^a at c. The income required to move the individual's budget line from Y^b to Y^c is the Hicksian *compensating variation*. Alternatively, we can ask what cut in income will hurt the individual as much as the price change does. In this case, we maintain the prices prevailing before the change in Fig. 2.2 and decrease income until the budget line becomes just tangent to U^b at d. The fall in income that moves the individual's budget line from Y^a to Y^d is the Hicksian *equivalent variation*.

These Hicksian measures are distinguished by their use of hypothetical changes in money income to duplicate the change in utility induced by the price change. The change in income required to compensate an individual for a price change is the compensating variation. The change in income that has the same effect on an individual as a price change is the equivalent variation. These measures are closely related but, as we shall see below, they are generally not identical.

²a, b, Y^a , Y^b , U^a , and U^b are the same as in Fig. 2.1.

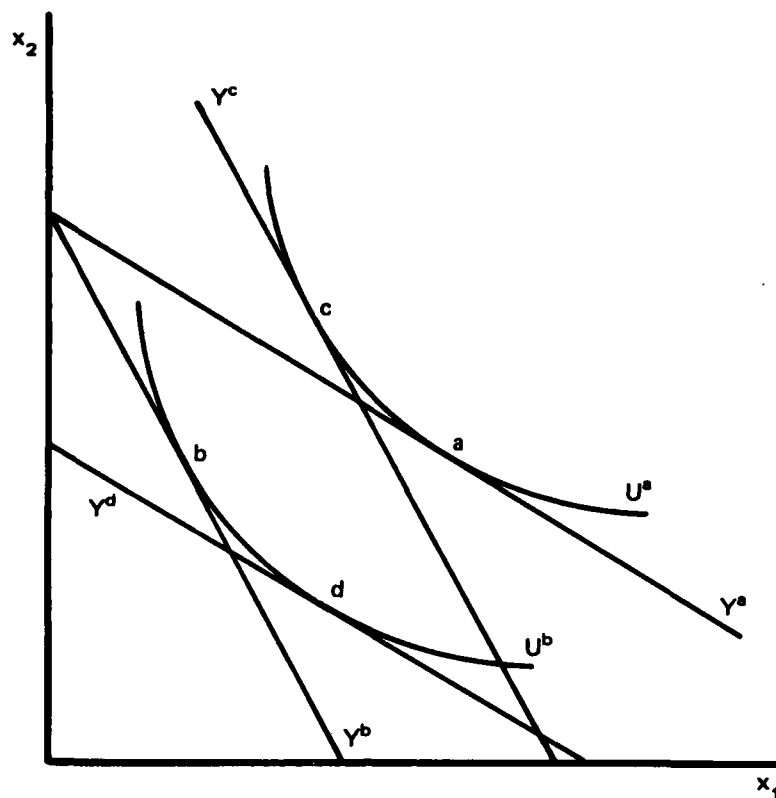


Fig. 2.2 — Two Hicksian measures of consumer surplus

The second way to measure the monetary effect of a price change on an individual, the "Marshallian," relates utility to money through the marginal utility of income, $\lambda = dU/dy$, which is the rate at which utility changes when money income rises incrementally. The inverse of this, $1/\lambda$, allows us to transform the change from U^a to U^b *directly* into monetary terms. Figure 2.3 illustrates this process. U^a shows the relationship between income (y) and utility at prechange prices; U^b shows the same relationship at postchange prices. U^a is uniformly higher because the price change reduces the feasible set for every level of income and hence reduces the achievable level of utility. The slope of each relation at each point, dU/dy , shows us how the individual relates incremental changes in utility and money income at that point. The price change moves the individual from a to b (which are fully analogous to a and b in Figs. 2.1 and 2.2).

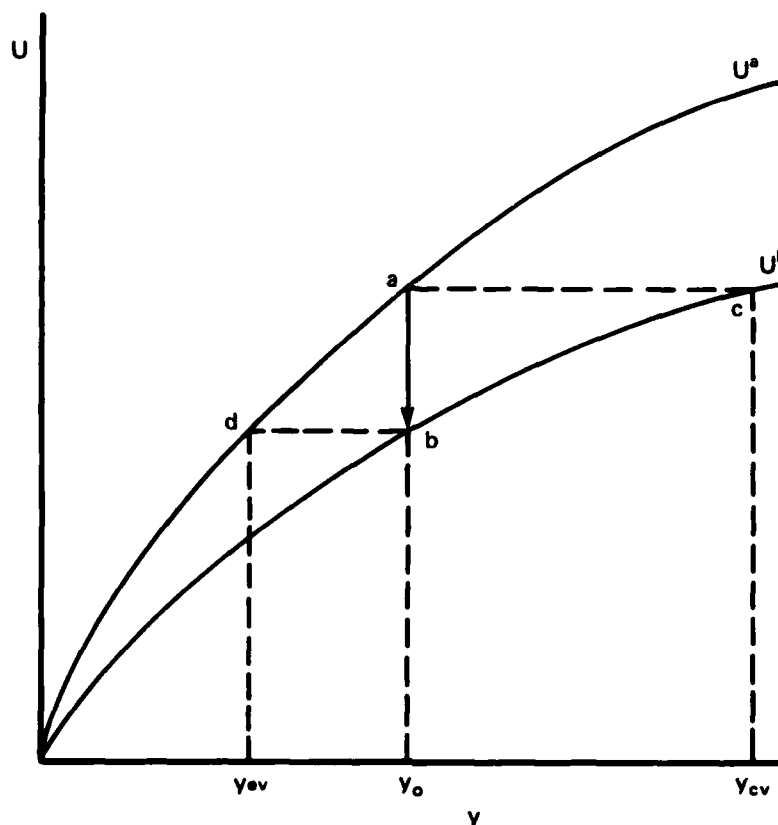


Fig. 2.3 – Utility-income relationships with Hicksian and Marshallian measures of consumer surplus

To measure mathematically the monetary effect of this price change on the individual, we must identify the "path" he uses to get from a to b . Since money income is fixed at y_0 , it is easiest to say he moves down the line segment marked with an arrow from a to b . This "movement" does not occur in a temporal sense; it occurs instantaneously. The path is meaningful only in the sense that the mathematics of maximization used in economics is based on infinitesimally small or "incremental" changes. To characterize the effects of a finite or "discrete" change, we must characterize that discrete change in terms of an infinitely large number of incremental changes. The path, then, represents a choice of incremental changes that we use to characterize the discrete change from a to b . Along the path shown from a to b , the monetary

value of each incremental change in utility is $dU/(dU/dy)$, where dU/dy is defined as the slope of each relationship between utility and income that intersects the path from a to b. As utility falls along this path, dU/dy changes. The trick in measuring the monetary equivalent of this change in utility is defining how dU/dy changes along this path.

Figure 2.3 also gives us a different way of thinking about the Hicksian measures. Increasing income to y_{cv} following the price change moves the individual from b to c, thereby compensating him for the price change; $y_{cv} - y_0$ is the compensating variation. Reducing income to y_{ev} before the price change moves the individual from a to d and is equivalent to changing the price from the individual's view of his utility level; $y_0 - y_{ev}$ is the equivalent variation.³

The third way to measure the monetary effect of a price change, Harberger's approach, is a combination of the first two. It essentially assumes that the revenues collected through the tax that induced the price rise, $t_1 x_1 = \Delta p_1 x_1$, are returned to the individual as a lump sum, $t_1 \bar{x}_1$. This effectively allows the individual to be compensated for the price rise, but not as much as the Hicksian compensating variation, the amount required to hold the individual unharmed. The individual's compensation is limited by the amount available within his initial budget constraint. This approach is a mix of the first two, then, in the sense that a Hicksian compensation is attempted, but it is constrained by the income constraint that ultimately underlies the Marshallian approach.

This procedure may at first sound unusual. It makes more sense if we address consumer surplus at the level of the economy as a whole, which is in fact what Harberger does. In this case, the level of compensation required by the Hicksian approach is constrained by the resources of the economy as a whole, represented in the budget constraint. On the other hand, the money collected in taxes under the Marshallian approach must go somewhere and, unless it is simply wasted or exported, it is probably returned to individuals as tax relief or services. In this setting, then, Harberger's approach seems quite

³Note that they can both be thought of as aggregations of $dU/(dU/dy)$ along paths of change, just as the Marshallian measure is. From this point of view, the Hicksian and Marshallian measures are trying to do very similar things.

reasonable. Here it is presented at the level of the individual instead of the economy both because Harberger's economy-wide compensation must ultimately find expression at the level of the individual and to maintain consistency across the approaches considered. Section IV aggregates consumer surplus across individuals for all measures.

Figure 2.4 illustrates this approach.⁴ Now, following the tax-induced price rise, the tax revenue collected is returned to the individual, changing his budget constraint from $(p_1^a + t_1)x_1 + p_2^a x_2 = y^a$ to $(p_1^a + t_1)x_1 + p_2^a x_2 = y^a + t_1 \bar{x}_1$. As \bar{x}_1 is raised from zero at b, the consumer moves along an income-expansion line, E, defined for prices $p_1^a + t_1$ and p_2^a . When E intersects Y^a , at e, \bar{x}_1 equals just the quantity of x_1 consumed and the individual's full tax payments are returned to him. The utility level at this point is U^e , which is tangent to Y^e at e by definition of E.

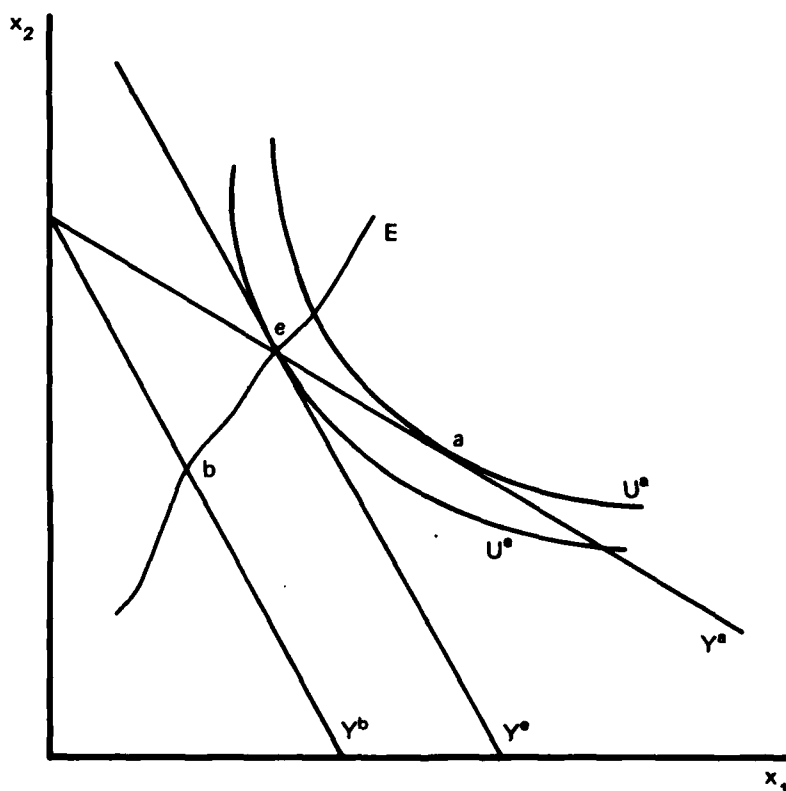


Fig. 2.4 – Harberger measure of consumer surplus

⁴ a , b , Y^a , Y^b , and U^a are the same as those in Figs. 2.1 and 2.2.

The Harberger measure seeks to monetize the difference between U^a and U^e . It does this just as the Marshallian approach does, by aggregating $dU/(dU/dy)$ between U^a and U^e and thereby finding the money income change that reflects this change in utility. In Fig. 2.3, U^e would lie between U^a and U^b .

CONSUMER SURPLUS WITH n GOODS

With these general descriptions as heuristic guides, let us now consider more carefully some algebraic expressions that allow us to measure these changes in utility numerically when an individual consumes more than two goods. Start by noting that consumer behavior is guided by a desire to maximize utility, in Eq. (2.1), subject to the budget constraint

$$\sum_{i=1}^n p_i x_i = y \quad (2.3)$$

The notion of consumer surplus is meaningless unless we believe that consumers maximize utility. If they do not, we cannot interpret observable data on prices in terms of a consumer's underlying willingness to pay. In particular, utility maximization assures that for any set of prices and income and $U_i \equiv \partial U / \partial x_i$,

$$U_i = \lambda p_i \quad (2.4)$$

where λ is the marginal utility of income.⁵ Changes in prices in Eq. (2.3), then, change the choice of goods consumed through Eq. (2.4), and the goods chosen ultimately affect utility. To trace the effects of price changes on utility, fully differentiate Eqs. (2.1) and (2.3).

$$dU = \sum_{i=1}^n U_i dx_i \quad (2.5)$$

$$\sum_{i=1}^n p_i dx_i + \sum_{i=1}^n x_i dp_i = dy \quad (2.6)$$

Hence, incremental changes in prices (and money income) affect goods consumed through the budget constraint by

$$\sum_{i=1}^n p_i dx_i = dy - \sum_{i=1}^n x_i dp_i \quad (2.7)$$

Further, Eq. (2.4) shows us how to choose among goods, so that

$$\sum_{i=1}^n p_i dx_i = \frac{1}{\lambda} \sum_{i=1}^n U_i dx_i = \frac{dU}{\lambda} \quad (2.8)$$

$$\frac{dU}{\lambda} = dy - \sum_{i=1}^n x_i dp_i$$

⁵We maximize a Lagrangian $U - \lambda(\sum p_i x_i - y)$. Equation (2.4) follows directly from the first-order conditions for this maximization. Equation (2.2) follows immediately from the inverse of a set of expressions $U_i/\lambda = p_i$. Hence we fully capture the demand relationships in Eqs. (2.2) through (2.4).

Equation (2.8) provides the basis for our three types of measures.⁶ Consider the three measures first in terms of *incremental* changes in prices and their incremental effects on well-being. Then use these incremental effects to determine how *discrete* changes in prices affect well-being. We do this by choosing sets of incremental price changes to represent discrete price changes--"paths" from prechange to postchange circumstances--and integrating across the incremental effects included in a chosen path to find the level of discrete effects on well-being.

CONSUMER SURPLUS FOR INCREMENTAL PRICE CHANGES

The Hicksian measures both attempt to hold utility constant: $dU = 0$. Hence, Eq. (2.8) becomes

$$dy = \sum_{i=1}^n x_i dp_i \quad (2.9)$$

Equation (2.9) shows *either* the amount of money income required to compensate for a set of price changes (compensating variation) or the change in money income that has the same effect on utility as the price changes (equivalent variation). For incremental price changes, these are identical.

The Marshallian measure seeks the monetized effect on utility of price changes when money income is held constant. Hence, $dy = 0$ and Eq. (2.8) becomes

⁶Equation (2.8) also allows a quick demonstration of why, as asserted earlier, λ is the marginal utility of income. For $dp_i = 0$, $\lambda = dU/dy$.

$$\frac{dU}{\lambda} = - \sum_{i=1}^n x_i dp_i \quad (2.10)$$

Though they use very different concepts, the Hicksian and Marshallian measures look identical in absolute value. We shall see that, except in extraordinary circumstances, this is true only for incremental departures from the set of prices prevailing before a tax is introduced.

The Harberger measure requires that any price change be offset by a change in income that returns tax revenues paid by the individual to the individual:⁷

$$dy = \sum_{i=1}^n d(t_i x_i) = \sum_{i=1}^n x_i dt_i + \sum_{i=1}^n t_i dx_i \quad (2.11)$$

Substituting Eq. (2.11) into Eq. (2.8) yields

$$\begin{aligned} \frac{dU}{\lambda} &= \sum_{i=1}^n x_i dt_i + \sum_{i=1}^n t_i dx_i - \sum_{i=1}^n x_i dp_i \\ &= \sum_{i=1}^n t_i dx_i + \sum_{i=1}^n x_i (dt_i - dp_i) \\ &= \sum_{i=1}^n t_i dx_i \end{aligned} \quad (2.12)$$

because $dt_i = dp_i$.

⁷If the tax reduces producer prices, the amount of tax paid by the individual is *defined* as $\sum (p_i - p_i^0) x_i$, where p_i^0 is the prefix level of the i th good. Hence, $dt_i = dp_i^0$. Recall that we assume here that producer prices are fixed.

Harberger's measure is quite different from the Hicksian and Marshallian measures. Note, however, that they allow no compensation *before* taking the measure. If we consider the first incremental change from a no-tax situation, $t_i = 0$ and $dU/\lambda = 0$ in Eq. (2.12). Under these circumstances, Eq. (2.11) tells us that the individual has received just the compensation called for by the Hicksian measure in Eq. (2.9). If such compensation had been provided before taking the Hicksian measure, no additional compensation would be required and the compensating and hence equivalent variation would be zero. In this sense, the Harberger and Hicksian measures are identical. Hence, for incremental departures from an initial set of prices, all three measures are essentially compatible.

CONSUMER SURPLUS FOR DISCRETE PRICE CHANGES

Differences emerge when we inquire into the effects of discrete price changes. Consider the Hicksian and Marshallian measures first. The Hicksian measures now become^{*}

$$\Delta y = \int_H dy = \int_{\Pi} \sum_{i=1}^n x_i dp_i \quad (2.13)$$

where H and Π indicate appropriate paths of income and prices, respectively. The Marshallian is

$$\Delta\left(\frac{U}{\lambda}\right) = \int_T \frac{dU}{\lambda} = - \int_{\Pi} \sum_{i=1}^n x_i dp_i \quad (2.14)$$

^{*}The integrals in Eqs. (2.13) and (2.14) are "line integrals." We will discuss how to evaluate them below.

where T is the appropriate path of monetized value of utility. The right sides of Eqs. (2.13) and (2.14) look identical but the integrals themselves take different values because different sets of x_i are involved, even if identical price changes are considered in each case. For example, Fig. 2.2 tells us that when measuring Hicksian compensating variation, we move from a to c for two goods. A fully analogous equivalent move is involved when we deal with n goods. For Hicksian equivalent variation, we move from b to d . Very different sets of x_i are involved even for these two Hicksian measures as the p_i change. Similarly, measurement of the Marshallian integral involves a movement from a to b and hence yet a third set of x_i . While the mathematical expressions look superficially alike, then, each will yield very different answers because each seeks to answer a different question.

We can use similar arguments to explain now why all three are equivalent for incremental changes away from an initial set of prices. In this case, a , b , c , and d effectively all coincide at a . Hence the levels of x_i are the same for all measures. This need not be true for incremental changes in prices away from this initial set of prices because in this case the x_i differ across each set of measures.

The Harberger measure, for discrete price (tax) changes, becomes

$$\Delta\left(\frac{U}{\lambda}\right) = \int_T \frac{dU}{\lambda} = \int_E \sum_{i=1}^n t_i dx_i \quad (2.15)$$

where E is the appropriate path of consumption levels for goods. This looks very different from the measures in Eqs. (2.13) and (2.14). Remember, however, that Eq. (2.15) reflects a kind of compensation not contemplated in the Hicksian and Marshallian measures. If taxes were not refunded, Eq. (2.15) would include the loss in taxes and would duplicate Eq. (2.14) in *mathematical* form. Nonetheless, the Harberger and Marshallian measures still diverge because the x_i considered in the two cases differ. As prices rise, the x_i (for normal goods) do not fall as fast in the Harberger case as in the Marshallian case because the Harberger measure in fact compensates the individual for at least part of his loss in well-being as prices rise, whereas the Marshallian does

not. Paradoxically, when we count taxes collected as part of the individual's loss in consumer surplus, Harberger's measure of loss is higher than the Marshallian measure because Harberger's compensation makes the individual value his lost consumption more than he would without compensation. This will become clearer in the next section.

INTEGRABILITY

The expressions in Eqs. (2.12), (2.13), and (2.14) all involve "line integrals," integrals over lines or paths through n-dimensional quantity or price space. Such integrals are "path-independent"--that is, they have unique values for discrete values of price changes--only if the integrand can be expressed as an exact differential of a well defined function. For example, consider an integral similar to that in Eq. (2.15):

$$\int_{\Xi} \sum_{i=1}^n \lambda p_i dx_i$$

Because $U_i = \lambda p_i$ (from Eq. (2.4)) and $dU = \sum_i U_i dx_i$ (from Eq. (2.5)),

$$\begin{aligned} \int_{\Xi} \sum_{i=1}^n \lambda p_i dx_i &= \int_{\Xi} \sum_{i=1}^n U_i dx_i \\ &= \int_{\Xi} dU = \Delta U \end{aligned} \quad (2.16)$$

Any discrete set of Δx_i will yield the same change, ΔU , no matter what path Ξ is chosen to effect Δx_i . This is because the individual starts from a well defined level of utility, U , before the change and ends at a well defined level. In this case, it is the levels we are interested in and not in how the individual got from one to the next--that is, in the set of incremental changes in x_i or path we chose to represent Δx_i .

The integrand in an arbitrary case,

$$\int \sum_i f_i(z_i) dz_i \quad (2.17)$$

will be an exact differential of a well defined function if and only if

$$\frac{\partial f_i}{\partial z_j} = \frac{\partial f_j}{\partial z_i} \quad \text{for all } i, j \quad (2.18)$$

The necessary condition in this statement will help provide some intuitive support for the statement. Suppose $\sum_i f_i(z_i) dz_i$ is an exact differential. Then

$$f_i = \frac{\partial f}{\partial z_i}, \quad \frac{\partial f_i}{\partial z_j} = \frac{\partial^2 f}{\partial z_j \partial z_i}, \quad \frac{\partial f_j}{\partial z_i} = \frac{\partial^2 f}{\partial z_i \partial z_j} \quad (2.19)$$

Under these circumstances, it must be true that Eq. (2.18) holds. Hence, Eq. (2.18) is a necessary condition for $\sum_i f_i(z_i) dz_i$ to be an exact differential.⁹ Let us consider Eq. (2.18) with regard to our three measures. In Eqs. (2.13) and (2.14), it becomes

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i} \quad (2.20)$$

⁹The sufficient condition is harder to establish. See, for example, Thomas (1960).

This will hold in Eq. (2.13) because a Hicksian measure by definition holds utility constant as prices change; therefore, the expressions in Eq. (2.20) are compensated cross price effects. One of Hicks's conditions on a compensated demand system requires that Eq. (2.20) hold. (See, for example, Allen, 1938, or Samuelson, 1965.) Hence, the integral in Eq. (2.13) is integrable.¹⁰ For Eq. (2.14), where money income remains constant as prices change, the expressions in Eq. (2.20) are uncompensated cross price effects. These are related to compensated cross price effects by the Slutsky equation:¹¹

$$\left. \frac{\partial x_i}{\partial p_j} \right|_M = \left. \frac{\partial x_i}{\partial p_j} \right|_H - x_j \frac{\partial x_i}{\partial y} \quad (2.21)$$

Equation (2.20) holds for Eq. (2.14) only if $x_j(\partial x_i / \partial y) = x_i(\partial x_j / \partial y)$ or, what is the same thing,

$$\frac{y}{x_i} \frac{\partial x_i}{\partial y} = \frac{y}{x_j} \frac{\partial x_j}{\partial y} \quad (2.22)$$

¹⁰Appendix A shows that $\int x_i dp_i$ is the exact differential of the individual's "expenditure function" and the measures that fall out of Eq. (2.13) are simply changes in the expenditure function.

¹¹We can derive this expression easily from the equations above. For a Marshallian demand function, $x_i = f(p_1, \dots, p_n, y)$,

$$\left. \frac{\partial x_i}{\partial p_j} \right|_H = \frac{\partial f}{\partial p_j} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial p_j}$$

where $\partial f / \partial p_j$ is the Marshallian price effect and $\partial y / \partial p_j$ is the change in income required to maintain the utility level when p_j changes. From Eq. (2.8), $\partial y / \partial p_j = x_j$. Substituting this into the expression above and rearranging terms yields Eq. (2.21) directly.

That is, the income elasticities for all goods *with changing prices* are equal. This can be true if *all* prices change only if the utility function is homothetic and hence all income elasticities are equal to one. In most cases, however, only a few prices change significantly and the income elasticities involved can take any value.¹² In general, income elasticities differ and hence Eq. (2.20) will not hold for Eq. (2.14). This means that the path of price changes matters. We will have more to say about this in Secs. IV and V.

Let us now turn to Harberger's measure in Eq. (2.15). For simplicity restate it as

$$\int_{\Xi} \sum_{i=1}^n p_i dx_i - \sum_{i=1}^n p_i^0 \Delta x_i \quad (2.23)$$

This expression is path-independent if

$$\frac{\partial p_i}{\partial x_j} = \frac{\partial p_j}{\partial x_i} \quad (2.24)$$

These are simply the cross price effects for inverse demand functions; Eq. (2.24) will hold under the same circumstances in which Eq. (2.20) holds. Because the x_i do not change in Eq. (2.23) to hold utility constant, Eq. (2.23) is not in general path-independent.¹³

How important is path independence? Its absence tells us that our integral does not measure movements along a well defined underlying function. As a result, no single value of well-being can be attached to a given set of prices (or quantities, depending on which space movements are defined in). In fact, with an appropriate series of price (or

¹²A common gambit in the theoretical literature is to set all elasticities but that for the numeraire equal to zero, thereby yielding "parallel" indifference curves. This is obviously of more theoretical than practical interest.

¹³For details, see Appendix C.

quantity) changes, *any* level of well-being can ultimately be associated with a given set of prices (or quantities).¹⁴ In a theoretical context, this is distressing because it raises questions about whether anything real is actually being measured by consumer surplus. This is probably the biggest reason why Hicksian measures currently dominate the measurement of consumer surplus. Hicksian measures are well defined and the underlying function they reflect--the expenditure function--is well understood.

Nonetheless, *given* a path, the Marshallian and Harberger measures have only one value. There is no ambiguity about this value. The Marshallian measure reflects actual marginal willingness to pay at each point along a path. When prices change over time, we can identify--or at least predict--a single price path and calculate single, reasonable values for Marshallian and Harberger consumer surplus. Only when prices change instantaneously does the concept of a price path become vague. In this case, consumers do not experience the continuous change in prices implied by a path. A path becomes an analytical artifact required to execute mathematical measures. In this case, simple restrictions on paths--for example, requiring that prices change monotonically from pre- to postchange levels--can be used to limit the range of potential values for Marshallian and Harberger consumer surplus (for example, see Willig, 1976). But within this range, economic theory gives us no basis for choice. The theory itself allows a basic indeterminacy in the amount consumers would be willing to pay to avoid an instantaneous price rise. As a general rule, this indeterminacy is larger, the larger the instantaneous change. Since price changes are more likely to occur over time as they grow in size, there are some natural limits on the extent of this indeterminacy. Nonetheless, for both Marshallian and Harberger measures, the choice of a projected path can be important in the measurement of consumer surplus. For Hicksian measures, we can use whatever path is computationally most convenient.

¹⁴For example consider two points, *a* and *b*, and two paths, I and II, for moving from *a* to *b*. Suppose the value of the integral of moving along the *i*th path from *a* to *b* is V_i and $V_I > V_{II}$. Then moving from *a* to *b* along I and from *b* to *a* along II leads to a value for the integral of $V_I - V_{II} > 0$. Continual cycling of this kind can increase (or decrease) the value of the integral to any level.

SUMMARY

An individual's utility is a function of the amount of goods he consumes. His choices about how much of each good to consume depend on prices and his income. Hence, when policies affect prices, he will change his consumption, thereby affecting his utility. Consumer surplus aims at capturing the effects of such price changes on utility in a monetary measure.

It does so by conceiving of discrete price changes as series of incremental price changes. When we assume that individuals maximize utility, economic theory allows us to determine the exact monetary effect on an individual of any incremental price change. By summing up such incremental effects, consumer surplus offers a discrete monetary effect of discrete price changes.

Unfortunately, the size of the discrete monetary effect we associate with a set of discrete price changes depends on the type of compensation we assume accompanies the set of price changes and the set of incremental price changes we choose to represent the discrete price change. Different assumptions about compensation lead to two different Hicksian measures--compensating and equivalent variation--and to Marshallian and Harberger measures that differ from these. The Hicksian measures assume two forms of full compensation. The Marshallian measure assumes no compensation. And the Harberger measure assumes a form of partial compensation. Different assumptions about the set of incremental price changes used to represent discrete price changes--that is, the "path" for moving from pre- to postchange prices--do not affect Hicksian measures. But they can affect Marshallian and Harberger measures; for these measures, information on the path of price changes can be helpful, especially when those price changes occur over time.

III. CONSUMER SURPLUS AND DEMAND CURVES

Consumer surplus is most often thought of as an area under a demand curve. As the last section emphasized, consumer surplus can be any of several monetary measures of the effects that price changes have on well-being. These measures can be thought of as areas under demand curves *only* if the proper areas and the proper demand curves are used in each case. The areas involved often bear no relation to demand curves as we traditionally think of them. This section explains how to translate the measures in Eqs. (2.13), (2.14), and (2.15) into geometric terms in price-quantity space. It first explains measures for changes in one price and then considers changes in many prices through a two-price example.

CHANGES IN ONE PRICE

When only one price changes, the evaluation of the integrals in Eqs. (2.13), (2.14), and (2.15) is simplified because we need not worry about the order in which prices change. There is only one path along which price changes can occur.¹ As a result, even integrals whose values are path-dependent have unique, well defined values in this case.

Equation (2.13) provides the basis for Hicksian measures. With only one good, Eq. (2.13) becomes

$$\Delta y = \int_{p_1^a}^{p_1^b} x_1 dp_1 \quad (3.1)$$

¹This will be true even if cross price effects are present and supply functions slope upward for substitutes, a case we consider in the next section. Several prices change at once in this case, but all changes can be linked to changes in a single variable--the policy change that precipitated the price change in the first market. Hence, a single price path is defined for all prices that change.

This expression takes on a specific value for any particular level of utility. For example, consider the level U^a in Fig. 2.2. Moving from a to c along U^a generates the Hicksian demand curve for x_1 shown by H^a in Fig. 3.1. This Hicksian demand curve defines the values of x_1 as p_1 moves from p_1^a to p_1^b in Fig. 2.2. Hence, Eq. (3.1), defined for U^a , is the area to the left of H^a between p_1^a and p_1^b . This area is the compensating variation. If, on the other hand, we are interested in values of x_1 along U^b , we take the area to the left of H^b between p_1^a and p_1^b . H^b lies to the left of H^a because it represents a lower level of utility than H^a . This area is the equivalent variation.²

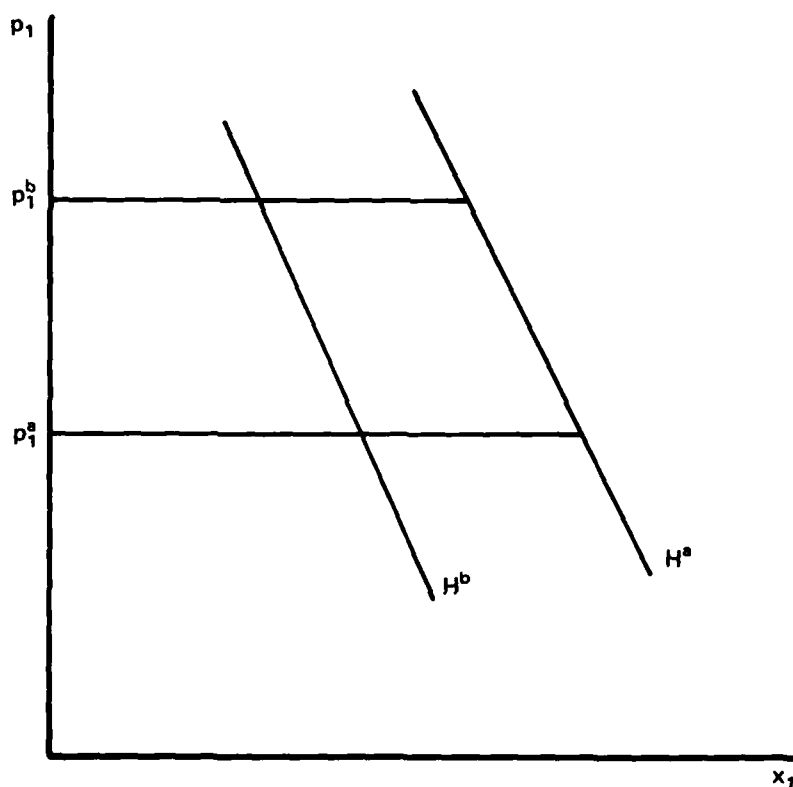


Fig. 3.1 — Hicksian compensating and equivalent variation

²When the price of one normal good changes, compensating variation exceeds equivalent variation if the price rises; the reverse holds if

With one good, the Marshallian measure in Eq. (2.14) becomes

$$\Delta\left(\frac{U}{\lambda}\right) = - \int_{p_1^a}^{p_1^b} x_1 dp_1 \quad (3.2)$$

The relevant movement in Fig. 2.2, from a to b, holds money income constant, not utility. The demand curve that describes how quantity reacts to price changes under this assumption is the Marshallian demand curve. Figure 3.2 superimposes this curve (M) on the Hicksian curves in Fig. 3.1. H^a and M intersect at a because at this price, the utility level underlying H^a and money income level underlying M are the same for both curves. As price rises above p_1^a , utility falls if money income remains constant (along M) or equivalently, money income must rise to maintain utility (along H^a). M falls faster than H^a as price rises because an individual is implicitly compensated along H^a and not along M. The higher level of income keeps his demand (for normal goods) higher on H^a than on M. The intersection of M and H^b can be explained in the same way. H^b lies to the left of H^a because it represents a lower level of utility and hence the individual consumes less of x, at any price along H^b than along H^a . The Marshallian measure uses the area to the left of M in Fig. 3.2. Note that this measure falls between the Hicksian measures in Fig. 3.2.

With one good, Harberger's measure becomes

$$\Delta\left(\frac{U}{\lambda}\right) = \int_{x_1^a}^{x_1^e} t_1 dx_1 = \int_{x_1^a}^{x_1^e} (p_1 - p_1^a) dx_1 \quad (3.3)$$

price falls. When the price of one inferior good changes, it can be shown that equivalent variation exceeds compensating variation if the price rises; the reverse holds if price falls. For a good with zero income elasticity, compensating and equivalent variation will always be equal.

This differs from the first two in two regards. First, it depends on yet another demand curve. Moving from a to e along Y^a in Fig. 2.4 traces out a Bailey demand curve for x_1 .³ Figure 3.3 superimposes it as

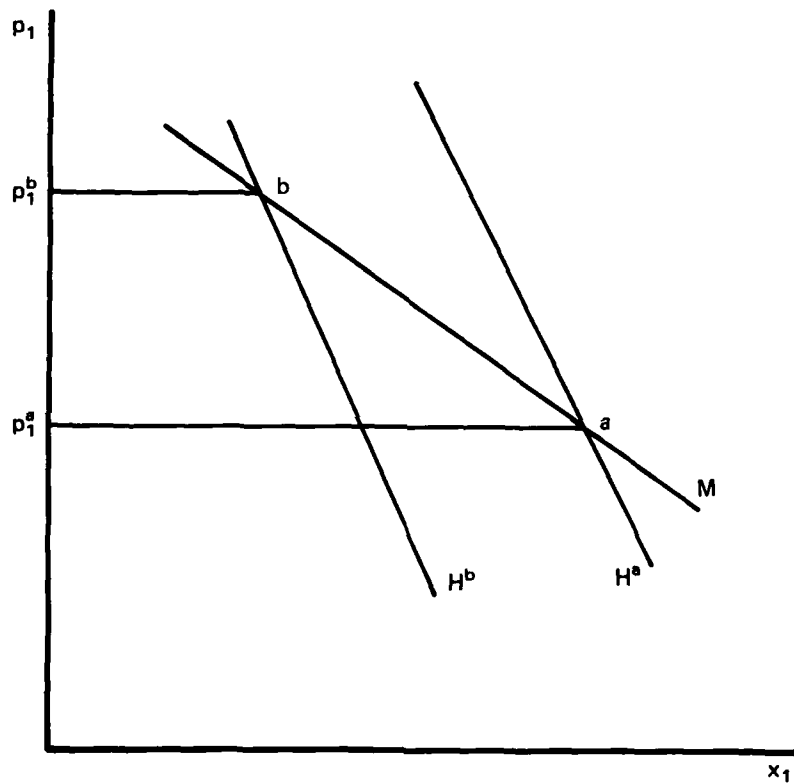


Fig. 3.2 – Marshallian and Hicksian changes in consumer surplus

³This demand curve is effectively defined by the marginal rate of substitution between the good in question and all other goods along a given resource constraint. That constraint is defined for the individual by his income constraint. See Bailey (1954).

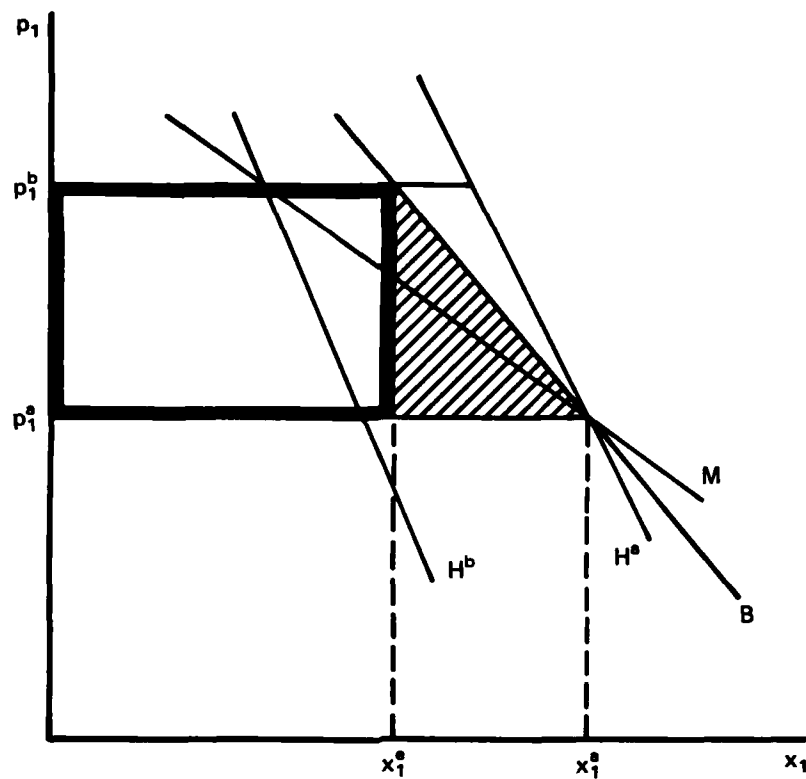


Fig. 3.3 — Harberger, Marshallian, and Hicksian changes in consumer surplus

B on the demand curves from Figs. 3.1 and 3.2. It falls between H_a and M because the form of compensation used tempers the effects of the price increase associated with the individual's income elasticity for x_1 while not compensating him enough to maintain the level of utility shown along U^a and H^a . The second reason why the Harberger measure is different is because it is made *after* this compensation, equal to the heavily outlined area in Fig. 3.3, is made. As a result, only the shaded area to the left of the relevant demand curve is measured. This is precisely the area included in the integral in Eq. (3.3).

This analysis of effects of changes in a single price tells us two important things. First and foremost, each measure uses a different *kind* of demand curve. Compensating and equivalent variations use two different Hicksian demand curves. The Marshallian measure uses a Marshallian demand curve. And the Harberger measure uses a Bailey

demand curve. The curves differ because of differences in assumptions about how money income changes as price changes; measures of consumer surplus differ for the same reason. Second, the areas in question are not really areas under demand curves. They are instead areas to the left of segments of demand curves. Empirical studies rarely give us good information on the values of demand curves outside the range of experience and, in particular, near the vertical axis. We do not need any information outside the range of price changes we are considering.

CHANGES IN MORE THAN ONE PRICE

When more than one price changes, evaluation of the integrals in Eqs. (2.13), (2.14), and (2.15) changes in two ways. First, more than one policy change may be involved. *Each* policy change must be dealt with in turn. Unfortunately, the order in which we consider policy changes--the price path we choose--can affect the numerical value of the relevant integrals. Second, any one policy change can affect areas measured in more than one market. Our assumption that supplier prices remain fixed simplifies this problem somewhat, but it can arise even here. We can break these problems down into those associated with integration across prices, in Eqs. (2.13) and (2.14), and those associated with integration across quantities, as in Eq. (2.15).

Consider integration across prices first. Suppose two taxes, t_1 and t_2 , are imposed, respectively, on x_1 and x_2 . Then the absolute values of Eqs. (2.13) and (2.14) become

$$\int_{p_1^a}^{p_1^{a+t_1}} x_1 dp_1 + \int_{p_2^a}^{p_2^{a+t_2}} x_2 dp_2 \quad (3.4)$$

As in the case of one price change, they have very different interpretations and involve different sets of x_i in the Hicksian and Marshallian cases. With more than one price change, we must decide in what order we should evaluate these integrals.

Let us suppose the tax on x_1 is imposed first and then the second tax is imposed.⁴ Figure 3.4 illustrates this case. Imposing the first tax moves p_1 from α to β in panel (a). Integrating over this change gives us the value of the first term in (3.4). It yields the outlined area in panel (b). As p_1 rises, the individual substitutes toward x_2 , causing his demand for x_2 to shift to the right, as shown in panel (c). Hence, when the second tax is imposed, moving us from β to γ in panel (a), the individual values the loss imposed by the tax along $x_2(p_1^a + t_1)$. We use this demand curve to evaluate the second term in Eq. (3.4). This yields the outlined area in panel (c). The sum of outlined areas in panels (b) and (c) gives us the value of Eq. (3.4) if the tax on x_1 is imposed first.

If the tax on x_2 is imposed first, the second term of Eq. (3.4) can be evaluated along $x_2(p_1^a)$, yielding the outlined area in panel (c) less the shaded area. This moves the individual from α to δ in panel (a). Turning to evaluation of the second tax, we find substitution toward x_1 induced by t_2 complete and hence evaluate the first term of Eq. (3.4) along $x_1(p_2^a + t_2)$. The individual moves from δ to γ in panel (a) yielding the sum of the outlined and shaded areas in panel (b).

For the value of the sum of integrals in Eq. (3.4) to be independent of the order in which taxes are imposed, the shaded areas in panels (b) and (c) must be equal. That is, we require that

$$\Delta p_1 \int_{p_2^a}^{p_2^{a+t_2}} \left(\frac{\partial x_1}{\partial p_2} \right) dp_2 = \Delta p_2 \int_{p_1^a}^{p_1^{a+t_1}} \left(\frac{\partial x_2}{\partial p_1} \right) dp_1 \quad (3.5)$$

⁴Keep in mind that the taxes are imposed simultaneously; to evaluate Eq. (3.4), however, we must impose some order. If the taxes are not imposed simultaneously, we can simply use the order in which they are actually imposed.

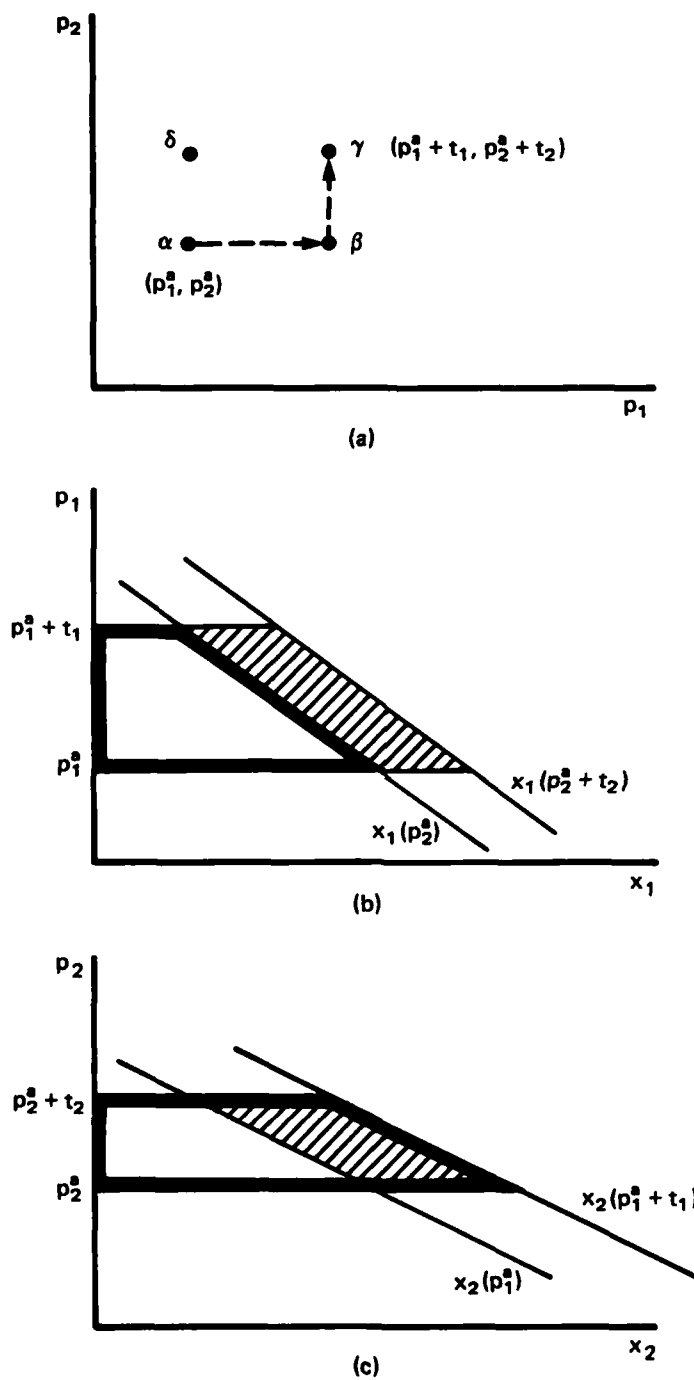


Fig. 3.4 — Change in consumer surplus when two prices change in sequence

If we linearize the demand curves, Eq. (3.6) becomes

$$\Delta p_1 \Delta p_2 \frac{\partial x_1}{\partial p_2} = \Delta p_2 \Delta p_1 \frac{\partial x_2}{\partial p_1} \quad (3.6)$$

That is, equality of the shaded areas in Fig. 3.4 is the basic notion underlying the cross price condition for integrability discussed in Sec. II. Recall that this condition is strictly satisfied only for Hicksian measures. Measures based on Marshallian or Bailey demand curves produce unequal shaded areas and hence depend on the order in which price changes are evaluated.

Suppose we could impose the taxes simultaneously. This could be done by making t_1 and t_2 functions of a single parameter, z , and rewriting Eq. (3.4) as

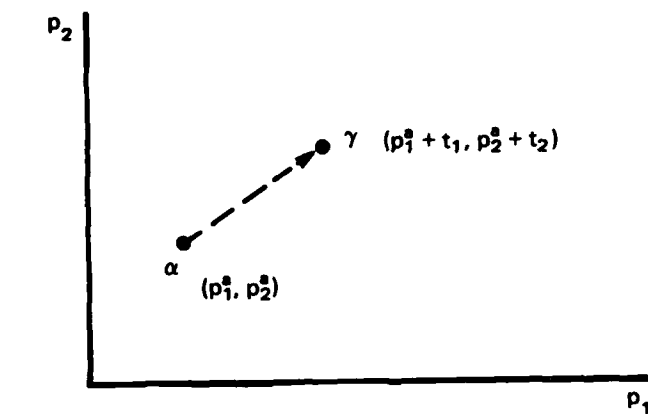
$$\int_0^1 \left[x_1 \frac{\partial t_1}{\partial z} + x_2 \frac{\partial t_2}{\partial z} \right] dz \quad (3.7)$$

where changing z from 0 to 1 just imposes both taxes.

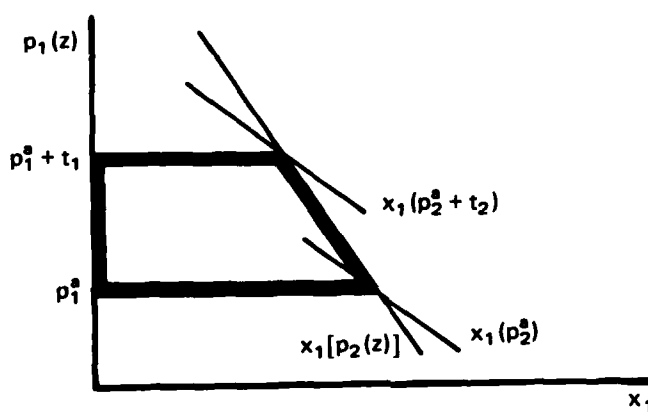
Figure 3.5 illustrates one way this might be done. As z moves from 0 to 1, the individual moves directly from α to γ in panel (a). The rising taxes move the individual not along demand curves, as we normally think of them, but along reaction curves in panels (b) and (c) for which levels of p_1 and p_2 change simultaneously.⁵ The sum of outlined areas in panels (b) and (c) provides the value of Eq. (3.7).

Because only one set of areas is possible, the value of Eq. (3.7) is unambiguous. This is because Eq. (3.7) is no longer a line integral and hence has only one value. But this does not allow us to escape the condition in Eq. (3.5). The value of Eq. (3.7) will change as the forms of $t_1(z)$ and $t_2(z)$ change. For example, the paths $\alpha\beta\gamma$ and $\alpha\delta\gamma$ shown in

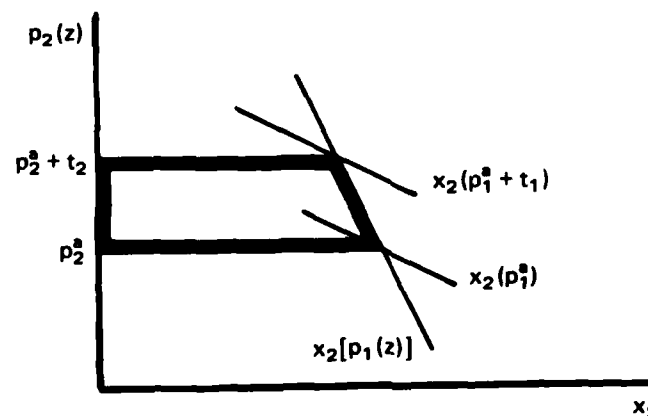
⁵Segments of the more traditional demand curves used in Fig. 3.4 are shown at initial points in Fig. 3.5. The individual begins in each panel on a traditional demand curve defined by $x_j(p_1^a)$ and ends on one defined by $x_j(p_1^a + t_1)$.



(a)



(b)



(c)

Fig. 3.5 — Change in consumer surplus when two prices change simultaneously

Fig. 3.4a simply reflect special forms of $t_1(z)$ and $t_2(z)$. Each parameterization can induce a different path from α to γ and, unless the integral in Eq. (3.4) is path-independent, each path can lead to a different value of consumer surplus.

Let us now consider the integration across quantities required by the Harberger measure in Eq. (2.15). This integration is more easily achieved if we convert it to a problem of integrating across prices. With changes in two prices, where p_1 changes first, Eq. (2.15) becomes

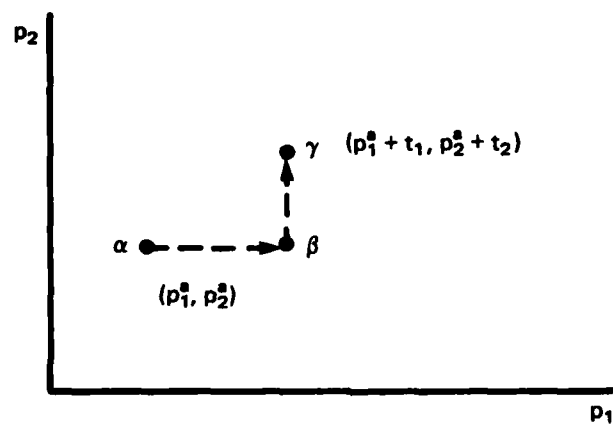
$$\int_{p_1^a}^{p_1^a + t_1} (p_1 - p_1^a) \frac{\partial x_1}{\partial p_1} dp_1 + \int_{p_2^a}^{p_2^a + t_2} \left[(p_2 - p_2^a) \frac{\partial x_2}{\partial p_2} + t_1 \frac{\partial x_1}{\partial p_2} \right] dp_2 \quad (3.8)$$

Equation (3.8) reflects the fact that

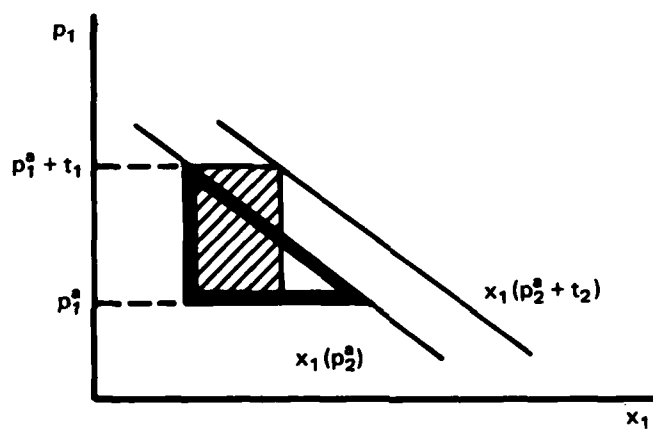
$$dx_1 = \frac{\partial x_1}{\partial p_1} dp_1 + \frac{\partial x_1}{\partial p_2} dp_2$$

$$dx_2 = \frac{\partial x_2}{\partial p_1} dp_1 + \frac{\partial x_2}{\partial p_2} dp_2 \quad (3.9)$$

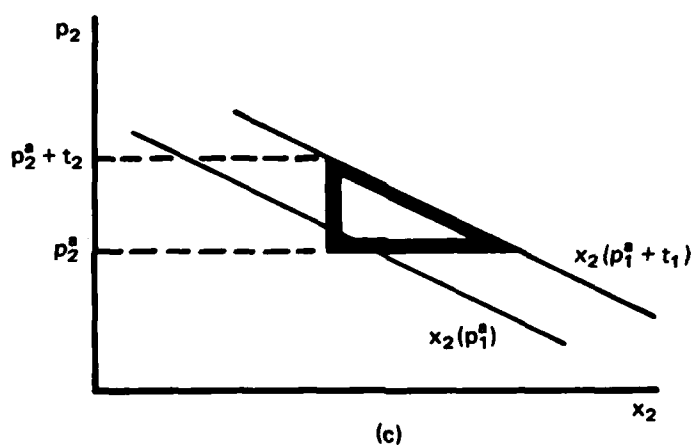
and $t_2 = 0$ under the integral when x_1 is taxed first. As above, imposing t_1 moves the individual from α to β in panel (a) of Fig. 3.6.



(a)



(b)



(c)

Fig. 3.6 — Harberger measures when two prices change in sequence

The first integral in Eq. (3.8) reflects the effects of this move; the value of this integral is shown by the outlined area in panel (b). Imposing t_2 , which moves the individual from β to γ in panel (a), has effects that must be measured in both markets. In panel (c), it induces a loss reflected by the outlined triangle under $x_2(p_1^a + t_1)$. This is the value of the first term of the second integral in Eq. (3.8). In addition, because a tax is in place on x_1 and the growing price of x_2 raises the individual's consumption of x_1 , the government collects additional tax revenues in the first market *and refunds them to the individual*.⁶ The losses reflected by the outlined triangles in panels (b) and (c), then, are offset by a tax refund equal to the shaded area in panel (b). The second term of the second integral in Eq. (3.8) provides this refund.

If we reversed the order in which taxes were imposed, triangles similar to those in panels (b) and (c) would measure losses from the taxes, but they would be measured under $x_1(p_2^a + t_2)$ in panel (b) and $x_2(p_1^a)$ in panel (c). The tax refund would now come on x_2 , since that is where a tax would be in place when the second tax was imposed. Hence, a shaded rectangle would appear in panel (c) bounded horizontally by p_2^a and $p_2^a + t_2$ and vertically by the outputs at $x_2(p_1^a, x_2^a + t_2)$ and $x_2(p_1^a + t_1, x_2^a + t_2)$.

If the demand curves in Fig. 3.6 were integrable, these areas together would indicate the same loss as the three areas in Fig. 3.6. Because these are Bailey demand curves, they are not integrable; effects on consumer surplus depend on the order in which taxes are imposed. We can impose the taxes simultaneously as we did in Fig. 3.5. Losses in consumer surplus would then be represented by triangles under reaction curves like those in Fig. 3.5 and no shaded rectangle would appear. Because Harberger uses Bailey demand curves, the sum of these triangles would yield yet a third estimate of the loss in consumer surplus, a priori as authoritative as an estimate based on any other parameterization of t_1 and t_2 .

⁶Because of the refund, the individual effectively pays p_1^a for a good he values at $p_1^a + t_1$ per marginal unit. It is because the individual values the good at a higher level than its effective cost to him that his increased consumption x_1 following the taxation of x_2 enhances his consumer surplus.

CONCLUSIONS

Consumer surplus can be related to demand curves, but we need a very broad view of demand curves to say that in general consumer surplus can be measured as the area under one or more demand curves.

To start, our three measures use three different demand curves. Only one of these--the Marshallian--is a behavioral function in the sense that it describes actual behavior in the absence of the forms of compensation envisioned in the other measures. Hicks and Bailey curves are not behavioral, unless of course compensation has occurred. We can infer their form from Marshallian curves. (See, for example, Appendix A.) The importance of the difference between behavioral functions and the demand functions used to evaluate consumer surplus will become more evident when we allow supply prices to change in response to policy changes in Sec. IV. In that case, even one policy change like a tax in one market can affect prices in many markets. Behavioral demand functions *must* be used to determine how prices and quantities are affected; Hicks and Bailey functions may be called into service only after these basic behavioral data are available to define the limits of integration in Eqs. (2.13), (2.14), and (2.15).

No matter what type of demand function is used, the demand curves based on prechange or postchange prices in other markets are *not* the curves relevant to measuring consumer surplus. Only when one price change is involved is information on demand curves defined by existing prices sufficient. Hence, no set of demand curves, as we traditionally think of them, in place at any time is appropriate to use for measuring changes in consumer surplus when more than one price changes. The effects of prices must be considered in turn and the demand curve relevant to any particular price change depends on which prices have already changed and which remain to be changed.

In this regard, we cannot attribute the loss reflected by a geometrical area in a particular market to that market itself or to the price change in that market. The size of this area will depend on where in the sequence of price changes a particular price change is considered. The loss in consumer surplus is the sum of areas in *all* markets and *only* this sum is a meaningful number for an individual.

Only it can be guaranteed to be independent of the sequence in which we consider price changes.

We can attempt to evade this discrete sequencing of price changes by parameterizing price changes so that prices change together. Such parameterization yields areas under reaction curves, not traditional demand curves in which the price in only one market is allowed to change. The curves that result are demand curves only in the sense that they show how much of a good is demanded at each point along a path of integration under the specific assumptions that define that path. If prices change over time, this can be an actual path, as under the assumptions for a Marshallian measure, or the hypothetical consumption path that would result if the conditions used to derive Hicksian and Bailey demand curves actually applied. If prices change instantaneously, one or more price paths must be assigned and the implications for consumption determined. In either case, it is probably better called a consumption locus than a demand curve.

The importance of the concept of a consumption locus as opposed to a demand curve becomes most evident in this section with regard to Harberger's approach. Rectangular "tax refunds" are simply areas under consumption loci. Such a locus in a particular market does not even reflect a price change in that market or any reaction to such a price change in that market. Calling such a locus a demand curve pushes that concept well beyond any bounds normally used to characterize demand curves. When we allow producer prices to change, such loci will become a routine part of not only Harberger's measure, but Hicksian and Marshallian measures as well.

With sufficient imagination, then, we can always associate consumer surplus with areas under demand curves. The imaginative reach required to do so, however, is probably more likely to lead to confusion and errors than to understanding. It would be more appropriate to associate consumer surplus with consumption loci. It would be more appropriate still to emphasize that *any* geometrical representation of consumer surplus is only a reflection of an explicit and well defined underlying concept. Any attempt to build and use geometrical analogs based on demand curves or consumption loci is likely to be successful only if it is tied carefully back to the underlying concept itself.

IV. CONSUMER SURPLUS IN THE AGGREGATE

We have seen that the concept of consumer surplus is based on revelations of individuals' willingness to pay for goods and services. Just as we can move from demand functions based on individuals' optimizing behavior to market demand functions, we can also move from individuals' consumer surpluses to aggregated forms of consumer surplus. Two important issues arise when we move beyond the individual. First, we move from a concept of individual welfare to one of social welfare, suggesting that measures of aggregate consumer surplus can help policymakers decide if public actions are "socially desirable" or not. Second, when we move to the market level, it is no longer appropriate to think of producer prices as being fixed. For example, a tax change in even one market can change both consumer and producer prices in many markets. This has important implications for the measurement of changes in consumer surplus. It also raises the possibility of changes in producer well-being, changes that can be represented through a concept very much like consumer surplus--producer surplus. We must understand the relationship between consumer and producer surplus before we can use consumer surplus to make statements about changes in social welfare. This section addresses these issues.

FROM INDIVIDUAL TO SOCIAL WELFARE

To understand consumer surplus, we first have to understand the maximization process that reveals an individual's willingness to pay for goods and the relationship of this willingness to pay to the individual's utility. Analogously, we must understand how policymakers view individuals' utilities to move from individual to "social" welfare. Traditionally, economists have used a "social welfare function" to do this. A social welfare function relates the utility levels of an individual in society to a single metric:

$$W = W(U^1, \dots, U^m) \quad (4.1)$$

where the j th individual consumes n goods:

$$U^j = U^j(x_{1j}, \dots, x_{nj}) \quad (4.2)$$

and the i th good is always totally consumed:

$$x_i = \sum_j x_{ij} \quad (4.3)$$

Who determines the form of this function is an important issue to the policy analyst. Economists have traditionally proceeded as though a society had a single social welfare function--for example, a function that embodied the tastes of a dictator. That is obviously an extreme view. Two alternatives will be more useful to policy analysts in a liberal democracy.

One simply says that each individual policymaker has his own social welfare function. In effect, the social welfare function is a formal way to embody a policymaker's views about the relative merits of individuals. This means of course that a measure based on one policymaker's view of the world may not be useful to another policymaker. That is, when we move beyond the individual, measures of changes in well-being cannot be value-free. That should not be surprising, but it potentially makes the task of extending consumer surplus beyond the individual in a useful way extremely difficult. At the very least, it says no single best aggregation of individuals' consumer surplus is possible.

The other starts from the premise that a liberal democracy assigns the task of moving beyond the individual to the political process. The

political process ultimately exists to define and maintain the social preferences of a group of individuals. If that process yields an orderly ranking of "alternative positions in which different individuals enjoy different utility levels"¹--and there is no guarantee that it does--then the policy analyst can potentially proceed *as if* a single social welfare function existed. Realized political outcomes can reveal information about that ranking in the same way that individuals' choices reveal information about the rankings that define their own utility functions. Hence, policy analysts can study political history to discover information about this social welfare function that they will need to aggregate the consumer surplus of individuals into a meaningful social metric.

For the time being, it will be useful to proceed as if many social functions are potentially relevant. Ultimately, the ideas of policy analysts are transformed into action by individual policymakers. If it is reasonable to hypothesize that a single social welfare function exists and is revealed through the political process, these policymakers may find information about it useful in predicting whether a specific policy option is viable--likely to survive the process. But they are at least as likely to have individual agendas that they prefer not to phrase in more general social or even political terms. In any case, it is safest for now to proceed as though many social welfare functions are potentially relevant. Section V will return to these issues.

Now suppose a set of price changes causes consumers to change their consumption patterns. How is social welfare changed? We can approach this problem just as we approached the individual's problem in Sec. II. We assume that individuals maximize their utilities. Just as in Sec. II, the very concept of consumer surplus is meaningless here without this assumption. From Eq. (2.4),

$$U_i^j = \lambda_j p_i \quad (4.4)$$

¹This comes from the definition of a social welfare function offered in Henderson and Quandt (1971, p. 282).

where $U_i^j \equiv \partial U^j / \partial x_{ij}$ and λ_j is the j th individual's marginal utility of income at y_j . Changes in prices change consumption, which changes utility, which finally changes social welfare. Start by looking at incremental changes. Totally differentiate Eqs. (4.1) to (4.3) and substitute from Eq. (4.4), using $W_j \equiv \partial W / \partial U^j$:

$$\begin{aligned} dW &= \sum_j W_j \sum_i U_i^j dx_{ij} \\ &= \sum_j W_j \lambda_j \sum_i p_i dx_{ij} \end{aligned}$$

or

$$\frac{dW}{\alpha} = \sum_j \frac{\alpha_j}{\alpha} \sum_i p_i dx_{ij} \quad (4.5)$$

for $\alpha_j \equiv W_j \lambda_j$ and α an average value of α_j . $\sum_i p_i dx_{ij}$ is simply our measure for the j th individual in Eq. (2.8). The change in social welfare is a weighted average of changes for individuals.

The weights, $\alpha_j = (\partial W / \partial U^j) / (\partial U^j / \partial y_j)$, represent the effect of adding an additional dollar to the j th individual's income on social welfare as a policymaker sees it. Given any distribution of income, all policymakers assign the same value of $\partial U^j / \partial y_j$ to the j th individual. But $\partial W / \partial U^j$ can and probably does vary from one policymaker to the next. As a result, we should expect policymakers to have different sets of weights for individuals. Or more reasonably, every policymaker has a different set of weights for *types* or distinct groups of individuals. If the policy analyst can discover what groups of individuals are relevant to a particular political debate, he can generate information on how policy affects each group. That is, so long as policymakers agree that all individuals in a given group should be equally weighted,

even if they disagree about the weight to be applied, the analyst can aggregate data on consumer surplus within this group. Policymakers can then apply their own weights, implicitly or explicitly, to such semiaggregated values to construct their own personal welfare metrics.

If it is costless to transfer income from one individual to another, policymakers will prefer a distribution of income in which an extra dollar transferred to one individual affects welfare in exactly the same way as an extra dollar transferred to another. If this were not the case, money could be transferred from one individual to another in a way that raised the policymaker's measure of social welfare.² In this special case, α_j is the same for every individual so that $\alpha_j = \alpha$ and Eq. (4.5) becomes

$$\frac{dW}{\alpha} = \sum_i p_i dx_i \quad (4.6)$$

which now brings our individual measure in Eq. (2.8) to the market level. If a policymaker is willing to accept the current distribution

²To see this, consider the problem of maximizing social welfare, $W(U^1, \dots, U^m)$ when $U^j = U^j(y^j)$ and $T(y_1, \dots, y_m) = 0$ defines the upper boundary on the set of feasible money incomes in society. The problem is to maximize $L = W[U^1(y_m)] - \phi T(y_1, \dots, y_m)$. The first-order conditions require, among other things, that

$$\frac{\partial L}{\partial y_j} = \frac{\partial W}{\partial U^j} \frac{\partial U^j}{\partial y_j} - \phi \frac{\partial T}{\partial y_j}$$

or for any two individuals i and j , using notation from the text and $T_j \equiv \partial T / \partial y_j$,

$$\frac{W_j \lambda_j}{W_i \lambda_i} = \frac{\alpha_j}{\alpha_i} = \frac{T_j}{T_i}$$

Holding all incomes but those of the i th and j th individuals constant, fully differentiating T yields $T_j/T_i = -dy_i/dy_j$. If transfers are costless, then dollar-for-dollar cash transfers are possible and $dy_i/dy_j = -1$. It follows that $\alpha_j/\alpha_i = 1$ or $\alpha_i = \alpha_j$. For a more complete discussion of this point and its application in a specific case where $\alpha_i \neq \alpha_j$, see Camm (1976).

of income as optimal in this special sense, then, changes in social welfare are likely to look to him very much like changes in consumer surplus as we have defined it for individuals. If not, he will need more information than aggregate demand levels in a market provide. Most policymakers are likely to reject such a view of optimality in the current distribution of income. Again, Sec. V returns to this issue.

AGGREGATE CONSUMER SURPLUS AND SOCIAL WELFARE WHEN PRODUCER PRICES REMAIN FIXED

Let us now consider how the concept of loss changes when we move from the individual to the social context. Start by holding producer prices fixed. This allows us to concentrate on consumers as we did in Sec. II. In a moment, we will examine the effects of allowing producer prices to vary.

Equation (4.5) provides a useful place from which to derive our Hicksian, Marshallian, and Harberger measures at the market level. The Hicksian measure attempts to keep each *individual's* level of utility constant. Hence, in Eq. (4.5) $dW = 0$ and

$$dy_j = \sum_i x_{ij} dp_i \quad \text{for all } j$$

We can use Eq. (4.3) to get

$$\sum_j dy_j = \sum_i X_i dp_i \quad (4.7)$$

where $\sum_j dy_j$ is the Hicksian measure (compensating or equivalent variation). Integration of the quantity on the right yields our measure for discrete changes. That is, the area to the left of *market* Hicksian demand curves provides an economy-wide measure of social gain or loss. Like the individual curves, the market curve is integrable because

$$\frac{\partial X_1}{\partial p_1} = \frac{\partial (\sum_j x_{1j})}{\partial p_1} = \sum_j \frac{\partial x_{1j}}{\partial p_1} = \sum_j \frac{\partial x_{1',j}}{\partial p_1} = \frac{\partial X_{1'}}{\partial p_1} \quad (4.8)$$

Note that the sum in Eq. (4.7) is independent of the weights discussed above. Individual consumer surpluses are equally weighted because the Hicksian measure implicitly accepts the utility levels associated with the existing distribution of income (before or after a policy change) and attempts to preserve them. No explicit interpersonal comparisons are required. Note, however, that the compensation contemplated in the measure must be distributed in a very specific way to maintain individual utility levels. Note also that the amount received by each individual, dy_j , accurately reflects the social opportunity cost of compensation funds; that is, transfers are costless.

For the Marshallian measure, $dy_j = 0$ for all j . From Eq. (2.6), we deduce that

$$\begin{aligned} \frac{dW}{\alpha} &= \sum_j \frac{\alpha_j}{\alpha} (dy_j - \sum_i x_{ij} dp_i) \\ &= - \sum_j \frac{\alpha_j}{\alpha} x_{1j} dp_1 \end{aligned} \quad (4.9)$$

Integrating over appropriate price paths yields our measure. The desirability of the distribution of income matters. We assign weights to individuals and sum the products of these weights with areas to the left of individual demand curves. For groups of individuals with equal weights, we can use areas to the left of group demand curves. Like the measure for individuals, this aggregate measure will generally not be path-independent.

Harberger's measure requires that Eq. (4.5) be adjusted to allow for compensation:

$$\frac{dW}{\alpha} = \sum_j \frac{1}{\alpha} \sum_i t_{ij} dx_{ij} \quad (4.10)$$

Areas of individual triangles and rectangles to the left of Bailey demand curves must be computed, weighted, and summed across individuals. In his work, Harberger explicitly rejects the use of weights, primarily because of the impossibility of economists agreeing on the values for weights, but also because their use can have unusual policy implications. (See Harberger, 1978.) Nonetheless, our analysis shows clearly that refusal to use weights requires us to accept the current distribution of income and assume that transfers are costless. When this is appropriate, we get the more familiar expression for Harberger's measure

$$\frac{dW}{\alpha} = \sum_i t_i dx_i \quad (4.11)$$

In general, the measure for discrete price changes is not path-independent at the individual or aggregate level.

AGGREGATE CONSUMER SURPLUS AND SOCIAL WELFARE WHEN PRODUCER PRICES CHANGE

Discussions of consumer welfare generally do not consider the fact that consumers also own factors of production. This is not important so long as producer prices remain constant. When producer prices do change, as they typically will when policies change, individuals are affected both as consumers and as producers. As a convention, effects on consumers and producers are typically treated as being separate. In the end, however, they are not. A simple extension of our analysis for the case of fixed producer prices demonstrates this clearly. After

presenting this extension, we will discuss its implications for our three measures.

The easiest way to show how changes in producer prices affect individuals is to recognize that individuals in a society act both as consumers and as producers through their ownership of productive factors. That is, an individual's income becomes endogenous:

$$y_j = \sum_k w_k z_{jk} \quad (4.12)$$

for w_k the wage paid the k th factor and z_{jk} the quantity of the k th factor that the j th individual owns. An individual's budget constraint is now defined by his fixed endowment of z_{jk}, \bar{z}_{jk} . Individuals sell these factors to firms that use them to produce the goods individuals finally consume.³ The i th industry has a production function

$$x_i = f^i(z_{i1}, \dots, z_{i\ell}) \quad (4.13)$$

and combines factors to maximize profits:

$$\pi_i \equiv c_i x_i - \sum_k w_k z_{ik} \quad (4.14)$$

where c_i is the producer price of the i th good. Maximizing Eq. (4.14) subject to Eq. (4.13) yields a basic profit maximization condition for a competitive industry:

³Individuals could also use factors to produce goods for their own consumption. This possibility does not change our basic results. We will not treat this possibility in detail.

$$w_k = \frac{\partial f^i}{\partial z_{ik}} c_i \quad (4.15)$$

Let us treat firms strictly as intermediaries so that any rents generated in production are passed back to the owners of factors of production. This will occur if firms all use linear homogeneous production technologies for which, by definition,

$$x_i = \sum_k \frac{\partial f^i}{\partial z_{ik}} z_{ik} \quad (4.16)$$

Then, substituting Eq. (4.15) into Eq. (4.16), we find that

$$c_i x_i = \sum_k w_k z_{ik} \quad (4.17)$$

where z_{ik} is the amount of the k th factor used to produce the i th good. Owners of factors receive the full revenues generated by the sale of the products their factors produce. Linear homogeneity also allows us to impute a quantity of output associated with any set of factors an individual owns. Since $\partial f^i / \partial z_{ik}$ does not depend on the owner of factor k , we can define

$$\hat{x}_{ij} \equiv \sum_k \frac{\partial f^i}{\partial z_{ik}} z_{ijk} \quad (4.18)$$

as the imputed amount of product i a factor owner produces when he sells z_{ijk} of factor k to the i th industry. Substituting Eq. (4.15) into Eq. (4.18) and summing across products yields

$$\sum_i c_i \hat{x}_{ij} = \sum_k w_k \sum_i z_{ijk} = \sum_k w_k z_{jk} \quad (4.19)$$

That is, we can define the individual's income in Eq. (4.12) in terms of the value of imputed production from the factors he owns. Substituting Eq. (4.19) into Eq. (4.12) and fully differentiating yields:

$$dy_j = \sum_i c_i d\hat{x}_{ij} - \sum_i \hat{x}_{ij} dc_i \quad (4.20)$$

Because z_{jk} are fixed for the individual, we can show that $\sum_i c_i d\hat{x}_{ij} = 0$. To see this, first fully differentiate Eq. (4.13) for the j th individual:

$$d\hat{x}_{ij} = \sum_k \frac{\partial f^i}{\partial z_{ik}} dz_{ijk} \quad (4.21)$$

From Eq. (4.15), note that for any k th factor and any two products, say the first and the i th,

$$\frac{\partial f^1}{\partial z_{1k}} c_1 = \frac{\partial f^i}{\partial z_{ik}} c_i \quad (4.22)$$

Hence, for any i th product, we can substitute Eq. (4.22) into Eq. (4.21) and find

$$c_1 \hat{dx}_{ij} = c_1 \sum_k \frac{\partial f^1}{\partial z_{1k}} dz_{ijk} \quad (4.23)$$

Summing across the j th individual's imputed production,

$$\begin{aligned} \sum_i c_1 \hat{dx}_{ij} &= c_1 \sum_k \frac{\partial f^1}{\partial z_{1k}} \sum_i dz_{ijk} \\ &= c_1 \sum_k \frac{\partial f^1}{\partial z_{1k}} dz_{jk} = 0 \end{aligned} \quad (4.24)$$

because z_{jk} are fixed by the j th individual's endowment. Using this result together with Eqs. (4.20), (4.5), and (2.8), then, tells us that

$$\frac{dW}{\alpha} = \sum_j \frac{1}{\alpha} \left(\sum_i \hat{x}_{ij} dc_i - \sum_i x_{ij} dp_i \right) \quad (4.25)$$

The second term in parentheses should be familiar from the last subsection as the basis for our Marshallian and Hicksian measures of consumer surplus; see Eqs. (4.7) and (4.9). The first term is an analogous term for losses in an individual's welfare as an owner of productive factors. It is the basis for measures of loss in *producer* surplus.⁴

As above, x_{ij} is a level of demand of the j th individual. \hat{x}_{ij} is effectively a level of supply and represents the distance from the vertical axis to the *individual's* imputed marginal cost curve for the

⁴When an individual produces for his own consumption, it will generally be true that $dc_i = dp_i$ because the good will not be taxed. Under these circumstances, that *portion* of the good produced for home consumption will net out and only the portion traded between individuals need concern us.

ith good. Integrating over incremental values of the producer surplus yields an area to the left of this marginal cost curve. In terms of income distribution, rents an individual earns from production are treated just like those earned from consumption. Weights are needed when transfers are costly, in the absence of an optimal distribution of income or--given that income is endogenous--an optimal distribution of factor endowments. When distribution is considered optimal, and transfers are costless, Eq. (4.25) becomes

$$\begin{aligned}\frac{dW}{\alpha} &= \sum_i \hat{X}_i dc_i - \sum_i X_i dp_i \\ &= \sum_i X_i (dc_i - dp_i) = - \sum_i X_i dt_i\end{aligned}\quad (4.26)$$

where \hat{X}_i is the *actual* market level of supply of the *i*th good.⁵

We can use Eq. (4.25) or Eq. (4.26) to calculate Hicksian and Marshallian measures. Hicksian measures hold *each* individual's utility constant by providing an exogenous infusion of money income to offset losses in the parentheses in Eq. (4.25). Hence, the incremental Hicksian measure becomes

$$\begin{aligned}\sum_j dy &= \sum_j \left(\sum_i x_{ij} dp_i - \sum_i \hat{x}_{ij} dc_i \right) \\ &= \sum_i X_i dt_i\end{aligned}\quad (4.27)$$

No weighting is required because the measure automatically maintains a given distribution of *utilities*. This expression remains integrable;

⁵Again, home production may be excluded without consequence since $dp_i = dc_i$.

therefore integration of Eq. (4.27) to yield discrete values of Hicksian measures generates unique values.⁶

The Marshallian measure uses Eq. (4.25) directly for incremental measures. Integration to yield a discrete measure also yields a path-dependent measure in most cases. Note that the producer surplus of this integral will be path-independent if $\partial \hat{x}_{ij} / \partial c_{i'} = \partial \hat{x}_{i'j} / \partial c_i$ for all i, i' . This is generally assumed to be true, either because in the absence of joint production, $\partial \hat{x}_{ij} / \partial c_{i'} = 0$ for $i \neq i'$, or with joint production because a well-defined production technology assures a well-defined cost function which in turn assures that the cross products above are equal. However, in general, individuals can withhold part of their factor endowment--for example, labor--for their own consumption, thereby making the amount of the factor supplied to firm an excess supply. Under these circumstances, factor supply represents something very much like a consumption decision. Hence, income levels and the marginal utility of income can affect how much a factor is valued and how much is therefore available for use in the market. Under these circumstances, Marshallian producer surplus is generally path-dependent.

Note that Eqs. (4.25) and (4.26) do not include revenues from the tax that precipitated the policy change. In the Harberger measure, this tax is explicitly returned to individuals. Hence the incremental Harberger measure becomes, under the assumptions of optimal income distribution and costless transfers,

⁶Note that we cannot use a traditional expenditure function to represent such integrals. An expenditure function makes sense only when all consumer prices equal producer prices. In this case, "consumer" surplus can be said to include both our consumer and our producer surplus measure. That is, *consumer* surplus includes areas that bear *absolutely* no relation to demand curves. See Appendix A. Because Eq. (4.27) is integrable, however, some underlying function can be identified to take the place of the expenditure function.

$$\begin{aligned}\frac{dW}{\alpha} &= -\sum_1 X_1 dt_1 + \sum_1 d(t_1 X_1) \\ &= \sum_1 t_1 dX_1\end{aligned}\quad (4.28)$$

This measure looks identical to that in Eq. (4.11). Remember, however, that in (4.28) producer prices can fall. The Harberger measure is more problematic when income distribution is not optimal or transfers are costly. Individuals must be compensated, by reference to the pretax consumer and producer price levels. When such compensation occurs, integration of Eq. (4.28) yields triangles and rectangles to the left of the relevant production and consumption loci. Figure 4.1 shows two such triangles for a simple case where x_{ij} and \hat{x}_{ij} are independent of other

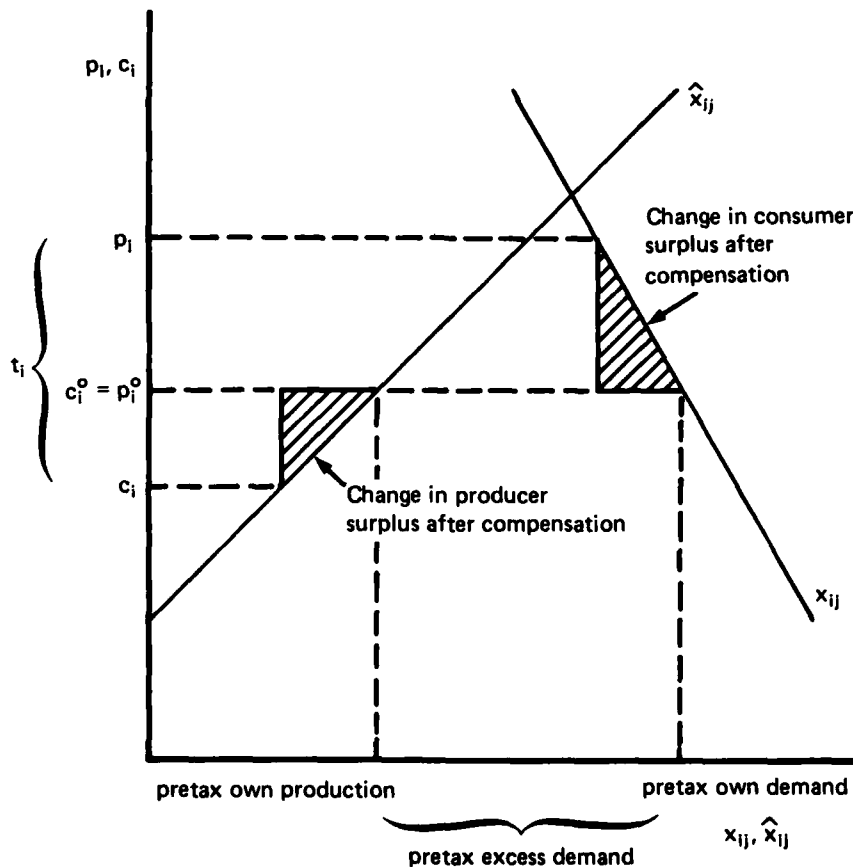


Fig. 4.1 — Harberger measures of consumer and producer surplus

prices. If $\alpha_j = \alpha$, we can sum these triangles across individuals to yield the single triangle most commonly associated with Harberger for such demand conditions. Recall that Harberger explicitly eschews weights.

Let us consider some more general geometric representations of changes in consumer and producer surpluses and tax revenues. For simplicity, we provide Marshallian measures and assume $\alpha_j = \alpha$. Figure 4.2 illustrates these measures for an *incrementally* increased tax that affects the prices of two goods; we start with a discrete tax in place on the first good and incrementally increase it. This incremental tax increase raises consumer price and lowers producer price incrementally for good 1. The increase in consumer price raises demand for its substitute, good 2, raising both consumer and producer prices by the same incremental amount for good 2. The rise in the consumer price of good 2 in turn raises demand for good 1. As the cross price effects work themselves out, the tax ultimately induces a new equilibrium, shown in panels (a) and (b). Consumer surplus falls by the sum of $X_1 dp_1$ (in panel (a)) and $X_2 dp_2$ (in panel (b)).⁷ From a social point of view, this loss is cancelled out in both markets: The government receives exactly $X_1 dp_1$ and producers of good 2 receive exactly $X_2 dc_2 = X_2 dp_2$. Meanwhile, producers of good 1 lose $-X_1 dc_1$, which is exactly made up, from a social viewpoint, by the government's gain of $-X_1 dc_1$. Finally, the government loses $-t_1 dX_1$, which no one receives. From a social point of view, this is the only net loss imposed by the tax. No tax revenue effects are shown in panel (b) because no tax is present in panel (b). If one were, Eq. (4.17) would pick it up just as it picks up $t_1 dX_1$ in panel (a).

Figure 4.3 extends these results to illustrate how *discrete* changes in consumer and producer surplus and social welfare can be related to market supply and demand functions. A discrete tax is imposed on good 1 where it did not exist before and its effects on the consumption and production of two goods are traced through. $X_1(p_1, p_2^a)$ and $X_2(p_2, p_1^a)$

⁷We need only deal with rectangular areas here, not trapezoids; the triangular areas shown vanish to zero when we speak of incremental (infinitesimal) price changes.

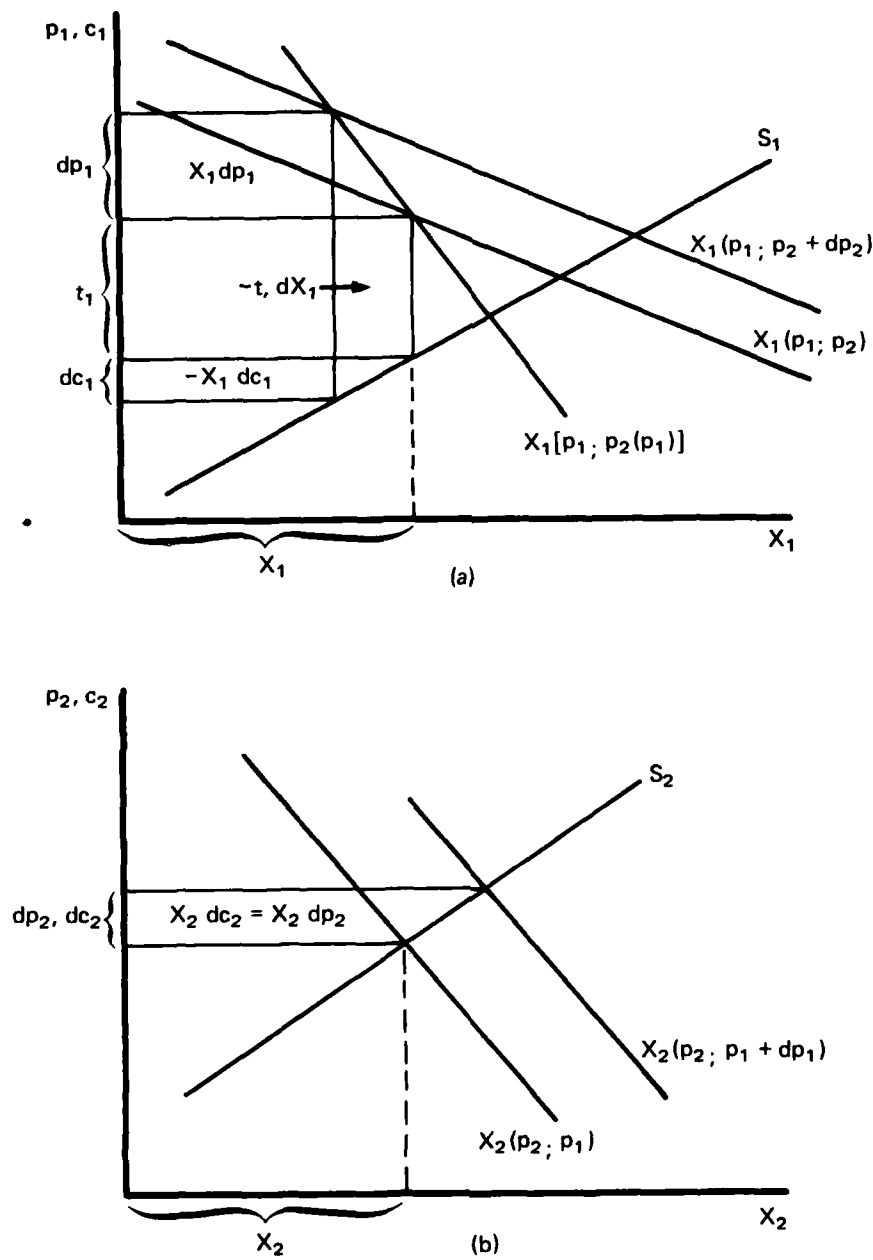


Fig. 4.2 – Changes in consumer and producer surplus when two prices change incrementally

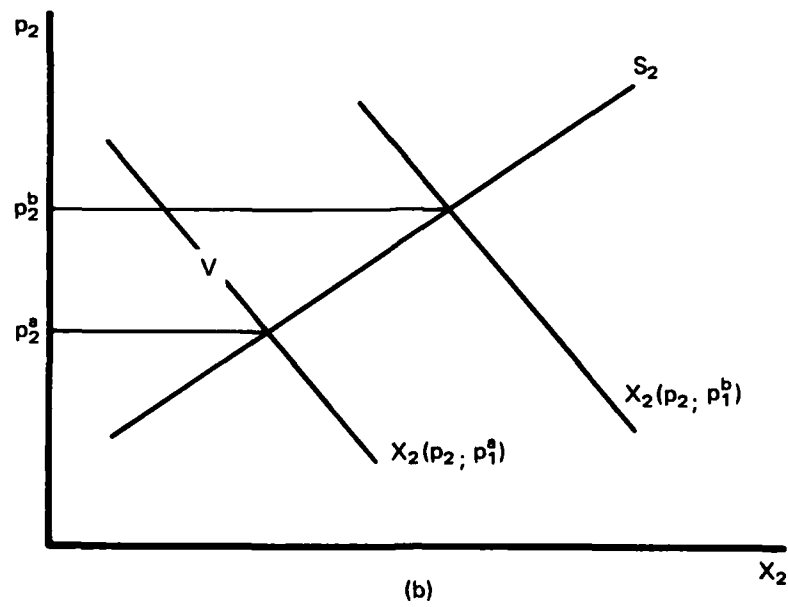
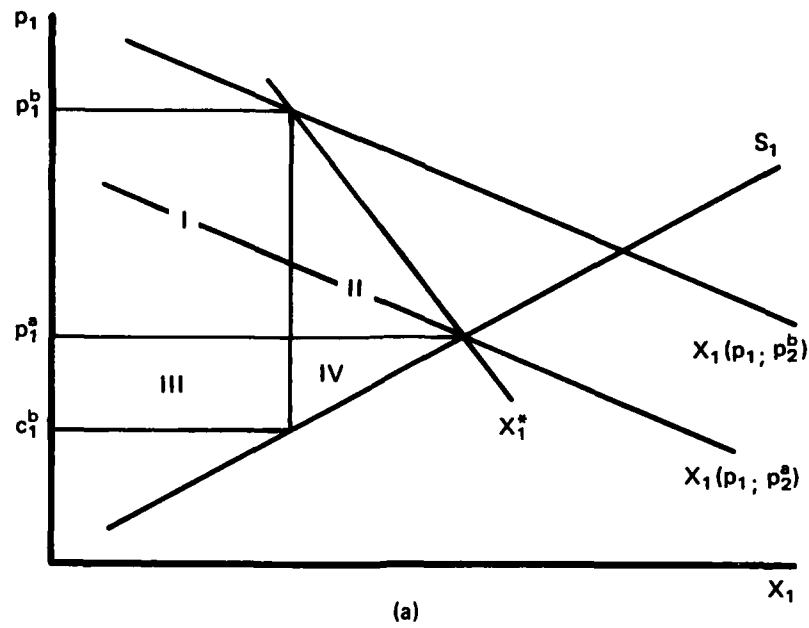


Fig. 4.3 — Changes in consumer and producer surplus when two prices change discretely

are the demand curves prevailing before the imposition of the tax; $X_1(p_1, p_2^b)$ and $X_2(p_2, p_1^b)$ are those prevailing after. (X_1^* is a reaction curve for good 1.) We find the following discrete changes by summing across incremental slivers like those in Fig. 4.2:

- Fall in consumer surplus: areas I, II, and V
- Fall in producer surplus for good 1: areas III and IV
- Rise in producer surplus for good 2: area V
- Rise in government tax revenues: areas I and III
- Net social loss: areas II and IV

To understand these results, note first that when a tax on one market raises the prices of substitutes, the taxed good will experience cross price effects and demand will rise at every price as the tax increases. That is, even in the simple case where only one tax is imposed, we cannot measure the loss it imposes on consumers by the area under a single demand curve *if* the tax affects the prices of substitutes. We must construct a reaction function like that labeled X_1^* in panel (a) of Figs. 4.2 and 4.1. Along X_1^{**}

$$X_1 = X_1[p_1(t_1), p_2(p_1(t_1))]$$

At each point along X_1^* , we use an incremental sliver like the one in Fig. 4.2 lying to the left of the demand function that applies for that specific level of the tax. We take the area to the left of a consumption locus, not a demand curve as we typically think of it.

*Empirically estimated Marshallian demand curves that do not properly control for the prices of substitutes may actually be reaction curves of this kind. For a discussion of how to construct such reaction curves from properly estimated demand curves, see Appendix B.

Which consumption locus should we actually use--that associated with Hicksian, Marshallian, or Bailey demand functions? Figures 4.2 and 4.3 use Marshallian curves. When tracing through cross price effects like those shown in Figs. 4.2 and 4.3, we must always start with Marshallian curves, even if we intend to use Hicksian or Harberger measures, unless we intend to implement Hicksian or Harberger compensation. Marshallian curves embody the behavioral functions that explain how a policy change actually affects prices without compensation. Then *given* the policy-induced changes in prices, we can calculate Hicksian or Harberger measures as functions strictly of the price changes. For Hicksian measures, we need only know the induced discrete price changes. Harberger measures are typically path-dependent, so the *path* of price changes may also be important. Those price changes can be translated into quantity changes as demonstrated in Eqs. (3.8) and (3.9). We use actual Marshallian consumption (and production) loci to compute prices; we then use hypothetical Hicksian or Harberger consumption loci to transform price changes into loss measures.

Note also that, while in Fig. 4.2b the incremental loss in consumer surplus can technically be measured as an area to the left of a demand curve, we will be hard pressed to say that discrete changes can be measured as the area to the left of a demand curve in Fig. 4.2b. Incremental slivers of the kind in Fig. 4.1 are summed in Fig. 4.2b to yield the shaded area to the left of the *supply* curve for X_2 . The relevant locus is not a supply curve or a demand curve per se but again a consumption locus. As the tax in panel (a) is incremented, we trace the locus in panel (b) and value the loss in consumer surplus from each incremental tax loss along the consumption locus. That locus lies along the supply curve because equilibrium moves along the supply curve as the tax rises.

It is important to keep in mind that these results are appropriate only if we accept the existing distribution of income (or factor endowments) as appropriate and assume transfers are costless. If we do not, effects on different individuals must be weighted and uncompensated transfers between individuals and the government need not net out as

they now do in Fig. 4.2. When weights are required, the expression in Eq. (4.25) puts heavy demands on the analyst. It requires the analyst to trace effects on "functional" incomes--incomes to owners of factors--through to effects on "personal" incomes--incomes to individuals who both consume products and own factors. While this is a desirable goal, it is rarely achieved in practice. Equation (4.25) can be broken up so that we consider effects on consumers and producers separately. To do this, we use Eq. (4.19) to define income so that the producer surplus component in Eq. (4.25) becomes

$$\sum_j \frac{1}{\alpha} \sum_k z_{jk} dw_k \quad (4.29)$$

(Keep in mind that z_{jk} is fixed.) Weights can then be applied to groups of individuals in their roles as consumers and to other, perhaps different groups of individuals in their roles as owners of factors. Section V discusses this possibility at more length.

SUMMARY

Any attempt to move beyond measures of the individual's consumer surplus to measures relevant to social decisions must inherently be value-laden. The values of individual policymakers can be embodied in weights they use to reflect the relative importance to them of changes in different individuals' consumer surplus. By restricting his activities to the measurement of changes in consumer surplus for individuals or groups of individuals likely to be viewed equally by most policymakers, the analyst can avoid value-laden arguments and concentrate on measures where price theory gives him a comparative advantage.

When producer prices remain constant during a policy change, aggregate measures of social welfare change simply consist of weighted sums of changes in the consumer surplus of individuals. For Hicksian measures, the weights are all equal to unity because these measures embody a commitment to the status quo that implies an acceptance of the

current distribution of income. Hicksian measures also neglect any cost of transferring funds. Weights make up an integral part of Marshallian and Harberger measures, although policymakers can choose to set these weights to unity. Harberger's argument for unitary weights as a professional standard does not satisfy the needs of policymakers unwilling to be indifferent about pure cash transfers among individuals.

When producer prices change during a policy change, aggregate measures of social welfare change must include measures of effects on producers as well as consumers. The structure of a social welfare function treats consumption and production simply as different activities by the same group of individuals treated when producer prices remained constant. That is, it emphasizes the personal distribution of income and argues that changes in individuals' well-being associated with their roles as producers should be treated the same way as changes associated with their roles as consumers: weighted in accordance with policymakers' views of social welfare and aggregated across individuals. Policymakers may find it easier to view consumption and production activities separately--to take a functional view of the distribution of income. We can break out a measure of the well-being of individuals associated with production--producer surplus--which policymakers can then treat just as they treat consumer surplus. To the extent that income effects are present in individuals' supply behavior, Hicksian, Marshallian, and Harberger measures will differ. Measures for individuals should be weighted and summed across individuals for Marshallian and Harberger measures. Acceptance of the status quo and the assumption of costless transfers implicit in the Hicksian measure call for unitary weights before aggregation.

These considerations raise serious doubts about the usefulness to most policymakers of unweighted aggregate measures of welfare change. Where weighted measures are appropriate, special doubts arise about Hicksian measures premised on the notion of exactly preserving the status quo.

V. COMPARING THREE MEASURES OF CONSUMER SURPLUS

To close, let us bring together the features of our three measures developed in earlier sections. A side-by-side comparison should help the policy analyst choose one measure over another when starting a study, or judge whether an existing study does or could give him the information he wants. This section starts by asking when the measures are likely to differ empirically and then compares their characteristics when it is clear that a choice must be made.

WHY MEASURES DIFFER

It should be clear now that each measure depends on a different concept of demand and these demand concepts differ *solely* because of income effects. In the absence of income effects, the three measures would be identical. That is, as a price rises, for a normal good, the quantity demanded falls faster along a Bailey demand curve than along a Hicks demand curve because the individual's money income is higher along the Hicks curve than along the Bailey curve. The same applies for comparisons between Bailey and Marshallian or Hicks and Marshallian curves.

Demand is higher on one than on the other because of a positive income elasticity. From Eqs. (2.21) and (C.10) in Appendix C, we can compare elasticities along the three curves as

$$\begin{aligned}\eta_{ij}^H - \eta_{ij}^M &= s_j \eta_{iy} \\ \eta_{ij}^B - \eta_{ij}^M &= s_j^* \eta_{iy} \\ \eta_{ij}^B - \eta_{ij}^H &= (s_j^* - s_j) \eta_{iy}\end{aligned}\tag{5.1}$$

where $\eta_{ij} \equiv (\partial x_i / \partial p_j) / (p_j / x_i)$, $\eta_{iy} \equiv (\partial x_i / \partial y) / (y / x_i)$, $s_j \equiv p_j x_j / y$, and s_j^* is a measure analogous to s_j , but more complex. Holding s_j and s_j^* constant, differences among the three measures all rise as η_{iy} departs more from zero. Holding η_{iy} constant, increases in s_j and s_j^* increase differences between the Marshallian and Hicks or Bailey measures. If s_j , s_j^* , and η_{iy} are all small, it is safe to say that differences in the way demand changes along different curves as prices change will also be small. In practice, s_j and s_j^* are very small for most goods. Only when we deal with aggregate services like housing that absorb large fractions of individuals' incomes or the supply of factors like labor that are important to income are these distinctions important empirically. It is common to suggest that such differences do not arise when we discuss supply curves. That is because income effects are not typically considered on the supply side of a market. In fact, the fraction of an individual's income likely to be affected by price changes is probably larger on the supply side than on the demand side. Treatment of the supply side would be fully analogous.

These statements about demand curves are important to us because all of our consumer surplus measures can ultimately be represented as areas to the left of consumption loci generated by these demand curves. If the demand curves are similar, the consumption loci and measures we derive from them will be similar as well. If there is any uncertainty about the locations of these demand curves, this uncertainty will make it impossible to distinguish different measures of consumer surplus empirically unless income effects are considerable. Willig (1973, 1976) has quantified the divergence of measures based on the Marshallian and Hicksian curves in terms of income elasticities and factor shares for arbitrary functional forms. Where these numbers are not large, his technique may prove useful in moving from one measure of consumer surplus to another.

One situation in which even small empirical differences among demand curves may be important is in the measurement of "deadweight loss," the net loss of social value induced by a new policy (Hausman, 1981). Deadweight loss is essentially the summation of all the

individual effects of a policy change when everyone is weighted equally. Because Harberger's form of compensation effectively internalizes all policy effects, the aggregate form of the Harberger consumer surplus measure with equal weights is equivalent to deadweight loss. But deadweight loss can be measured with Marshallian and Hicksian, as well as Bailey, demand curves. Figure 5.1 presents a case where a tax affects the price of only one good and supply price remains unperturbed. The difference between the Hicksian compensating variation ($I + II + III + IV + V$) and Marshallian change in consumer surplus ($I + II + V$) is $III + IV$, a value that is likely to be small in relation to either of these unless the two demand curves differ dramatically. The difference between the Hicksian ($IV + V$) and Marshallian ($V + II$) measure of deadweight loss is $II - IV$; this difference can be substantial in relation to either of these measures. When our primary interest is this

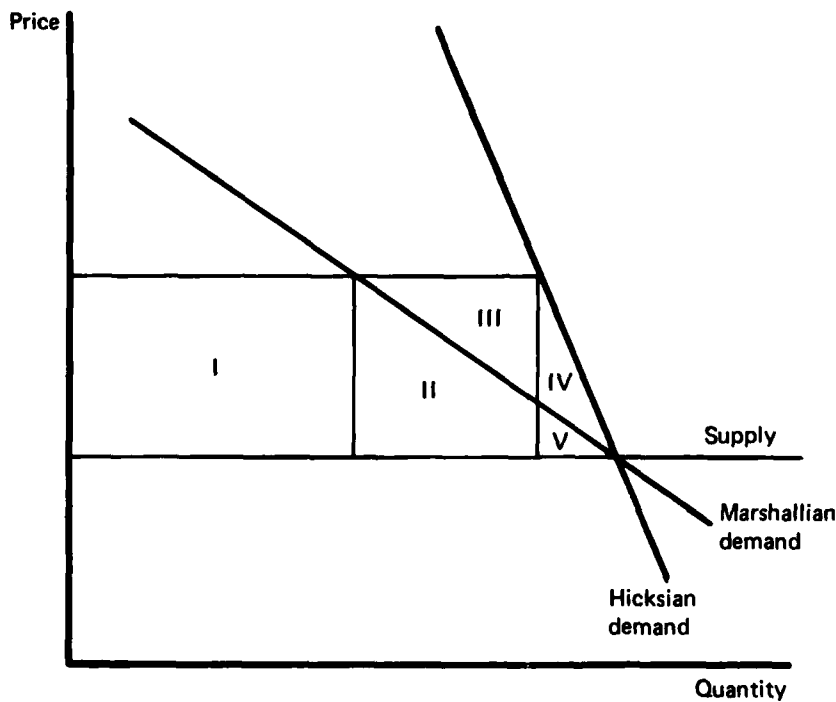


Fig. 5.1 — Marshallian and Hicksian measures of deadweight loss

final summation of all effects--the net social or deadweight loss from a tax--a comparison of alternative measures of welfare loss and their magnitudes deserves special attention.

THE ROLE OF BEHAVIORAL FUNCTIONS

Of the three demand functions we have considered, only the Marshallian characterizes true behavior unless compensation is in fact expected. Hicks and Bailey functions are hypothetical functions that must be inferred from the Marshallian functions we can observe directly in the absence of compensation. This has two important implications for comparing measures.

First, if price changes cannot be inferred directly from policy changes without reference to market data, we must use Marshallian functions to calculate the price changes relevant to our welfare measures, no matter which measure we ultimately choose. Marshallian functions tell us how policy has shifted the market equilibrium. We can then take the price changes implied by this change in equilibrium and calculate any measure we desire. This applies on the consumption side of the market as well as the production side, where measures differ on that side.

The second implication is more subtle. Just as a Marshallian function reflects actual choices at each point along a continuum, Marshallian consumer surplus reflects actual valuation along a continuum. In this sense, it is a more realistic measure of value--willingness to pay--than the Hicksian or Harberger measures. To see this, consider the following exercise. Suppose an individual faces the prospect of a small rise in one price. All of our measures of willingness to pay to avoid that price rise give the same answer, because all demand functions yield the same level of consumption at this point. Now suppose the individual faces a slightly higher rise in the same price. The initial rise lowers his real income, thereby lowering this willingness to pay to avoid any further price increase. The individual will realize this not only following the initial increment, but also before it happens. Hence, unless he expects to be compensated,

his willingness to pay to avoid the total price rise will be reflected by the Marshallian measure. He can anticipate price effects on his valuation of income and reflect them in his willingness to pay, no matter how large the price change involved. A similar argument holds equally well for reductions in price; the individual can anticipate that price reductions increase real income and thereby increase his willingness to pay for a price reduction, no matter how large it is.

If we are interested in a measure based directly on willingness to pay, then, Hicksian and Harberger measures make sense only if the hypothetical compensation they posit is actually anticipated. If it is, then the demand functions underlying these measures become behavioral functions. They become valid *only* because they are behavioral functions. Given that direct and complete compensation rarely occurs, it is hard to imagine Hicksian functions as behavioral.

Bailey curves are more likely to be behavioral in the following sense. An individual can often expect a tax to be collected and spent on something of value to him. Hence, taxes need not be simple reductions in wealth. The goods they make possible will tend to increase demand for other goods as well, inducing what appears to be a form of monetary compensation with regard to any particular good. What this suggests is that, although the exact form of compensation envisioned in the Bailey function may not be appropriate, the general notion of partial compensation is. Hence it may be entirely appropriate to think of demand functions *like* the Bailey function. To the extent that distinctions among functions are empirically important, it may be appropriate to determine precisely which function best reflects this form of implicit compensation. In the end, there are as many different functions we can use as a basis for consumer surplus measures as there are forms of implicit compensation. This suggests that policies that lead to identical changes in prices can have very different effects on willingness to pay to obtain or avoid them.¹

¹Silberberg (1972) has argued that there are as many measures of consumer surplus as there are price paths. We suggest something

In this regard, the Harberger measure *may* be more acceptable in more aggregate applications. The narrower the class of individuals represented in a Harberger measure, the more likely that actual compensatory effects will depart from those envisioned in the Bailey demand function. As more individuals are included, the idea that rents extracted (or transferred) have to go (or come from) somewhere else within the same group of individuals becomes more compelling. In fact, this seems to be the key idea behind Harberger's dependence on Bailey functions. This is valid, of course, only so long as weighting is not required.

THE IMPORTANCE OF INTEGRABILITY

Only Hicksian measures are path-independent in all cases. This fact is all important to analysts seeking a well defined, well behaved welfare indicator. Only with path-independence can we argue that consumer surplus directly represents some underlying function. In particular, the equivalent variation is the preferred Hicksian welfare indicator. That is because it maps any set of price changes into a single scalar equivalent value: the change in money income with the same effect on utility.² It may seem paradoxical that, although Marshallian and Harberger measures of consumer surplus explicitly seek to measure changes in the monetary value of welfare, $\int dU/\lambda$, only Hicksian measures that make no explicit use of marginal utility of income in fact produce well defined monetary values of changes in welfare. Four points are important.

different here. Even for identical price paths, the rents extracted from consumers by any set of price changes can be used in many ways and the ways in which they are used will affect the consumption levels of the goods whose prices have changed. This change in consumption changes the measure of consumer surplus. An analogous argument can be made about the sources of rents when price changes benefit consumers.

²It is worth noting that in comparing two options, A and B, the equivalent variation associated with moving from A to B equals the compensation variation associated with moving from B to A. The equivalent variation is preferred basically because it allows us to use prechange prices to measure the effects of any change and thereby yields a complete ranking based on a single set of prices. For details, see Hause (1975).

First, integrability becomes an issue only when policy changes induce several prices to change at once. When only one price changes, all three measures can be calculated with a simple integral. Hence, all three are well defined.

Second, even when many policies change, Marshallian and Harberger measures are well defined along any particular path of prices. In particular, they are well defined along the path that actually occurs or is expected to occur when price changes occur over time. That we cannot identify an "indirect welfare function," in which the monetary value of welfare is a function of prices and income, should not suggest that the Marshallian and Harberger measures do not measure something real and meaningful. As we saw above, the Marshallian concept measures *actual* willingness to pay along any price path. That willingness to pay to obtain or avoid any set of discrete price changes is path-dependent is a reflection of the fact that the individual will be willing to pay different quantities of money depending on the order in which changes are made. There is nothing mysterious about this. It simply says that if we care about actual willingness to pay, we need to know more than the discrete change in prices contemplated; we also need to know the path over which prices will change. The Harberger measure can be explained in the same way so long as it represents a behavioral function.

Third, when many policies change simultaneously and instantaneously, actual price paths are discontinuous. Mathematical measures of consumer surplus are impossible without continuous price paths; hence, we must choose arbitrary continuous price paths to fill the real price discontinuity. Any path will do for Hicksian measures: Because they all yield the same value for a change in consumer surplus, we need only choose the most convenient one. Different paths give different values for Marshallian and Harberger measures. If we are willing to consider only monotonic price paths--paths along which prices move monotonically from their prechange to their postchange levels--we can use well defined Hicksian measures to bound these. The geometric analogs discussed in Secs. III and IV essentially do precisely that. The measures that result are intuitively satisfying in the sense that their rankings are consistent with the forms of compensation that underlie different measures of consumer surplus. In the end, a residual

theoretical uncertainty inevitably resides in Marshallian and Harberger measures. When several prices change instantaneously, economic theory offers no unique values that we can attach to the effects of such price changes unless we specify a very specific form of compensation. To the extent that these measures of consumer surplus differ empirically, however, an approximate idea of the size of the proper Marshallian or Harberger measure can be more useful to policymakers than an exact measure of a Hicksian version of consumer surplus when Hicksian compensation is not expected.

Fourth, the monetary value that a Hicksian measure assigns to a change in the level of welfare is inexorably linked to only one point that actually occurs. For compensating variation, it is the prechange status quo. It represents the amount of compensation that must be made to maintain an individual's prechange utility level. That is potentially important if compensation is actually going to be made. But if it is not, what does the measure mean to a policymaker? To the extent that consumer surplus has any meaning to him, it is as a measure of how individuals feel a policy actually affects them. From this point of view, compensating variation is a hypothetical quantity with no immediate importance to the actual policy process.

Equivalent variation is linked to the postchange situation just as compensating variation is linked to the prechange status quo.³ It is a hypothetical construct with no immediate intuitive meaning for the policymaker. One might say that it is the change in money income that has the same effect as a change in prices. But if that money income measure is meaningful in a policy context, why do we observe the following peculiarity: If we move from policy option A to B and then from B to C, the money equivalent associated with moving from A to C is not the sum of the money equivalents of moving from A to B and then from B to C. Figure 5.2 illustrates this for a case where policy changes affect the price of only one good.⁴ Moving from A to C yields an

³This is a reflection of the relationship of these two measures suggested in footnote 2.

⁴ H_1 are Hicksian demand curves; M is the behavioral Marshallian curve. Cf. Winch (1965).

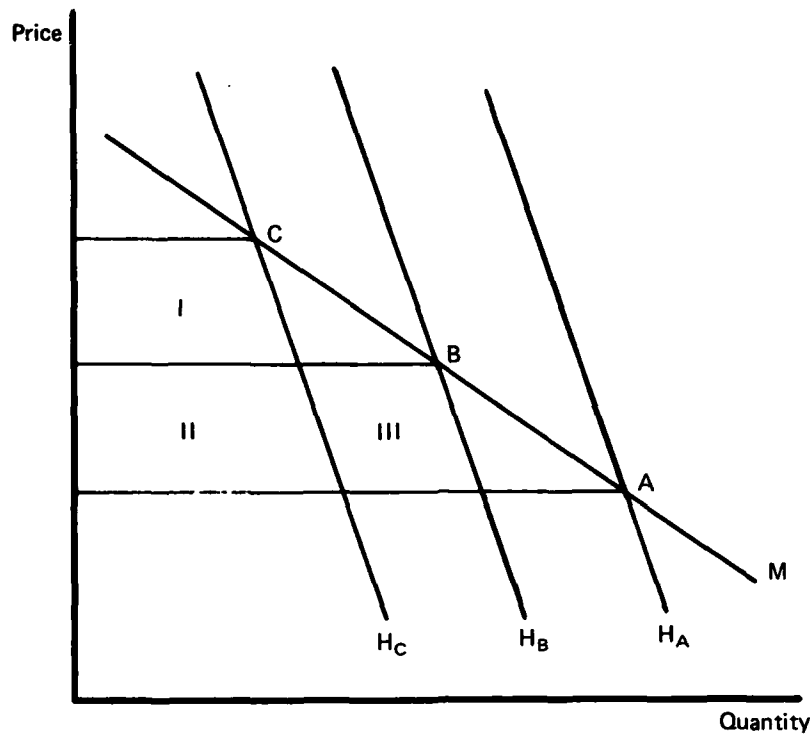


Fig. 5.2 — Nonadditivity of Hicksian measures when prices change in steps

equivalent variation equal to the sum of areas I and II. For a move from A to B, the relevant areas are II and III; for B to C, we use area I. Hence the separate moves yield a monetary equivalent equal to areas I, II, and III. The direct move includes only areas I and II. Intuitively, this looks like a terrible case of path-dependence, though mathematically we know it is not. Repeating the exercise for a Marshallian measure yields no such difficulty.

This will present the most serious problem to policymakers when B and C represent alternatives to the status quo at A that they are currently considering. Using the equivalent variation measures of loss of $II + III$ for alternative B and $I + II$ for alternative C, the additional loss of moving from B to C, when we currently stand at A, is $I - III$. This incremental loss could conceivably be negative, suggesting that consumers are willing to pay to have a policy alternative with a higher price. More generally--and more likely--such a comparison will understate the increase in cost to consumers of

choosing C instead of B. This is true whether prices change over time or instantaneously when we move from A to B or C.

The point here is simply that, although Hicksian measures yield what many economists consider desirable welfare indicators, these welfare indicators are not so desirable for the policymaker because they do not represent quantities with direct applications in the policymaking arena as it exists today. Hence, the underlying welfare function that integrability grants to Hicksian measures may not be relevant to policymakers.

AGGREGATE AND DISTRIBUTIVE MEASURES

When transfers are costless and policymakers agree that the distribution of income is optimal, we have seen that we can weight everyone's monetary willingness to pay equally. This is because policymakers treat everyone equally and are indifferent to transfers from any one individual to another. It suggests that only aggregate measures of changes in consumer (and producer) surplus are necessary. It also suggests that the current distribution of income should be preserved, giving special cogency to the Hicksian and Harberger measures that reflect forms of compensation that preserve the status quo.

However, even if everyone accepts the status quo and agrees to decide a particular policy issue on the basis of aggregate measures, not everyone in a society is equally affected by any policy change. The very notion that allows us to use an aggregate figure--support of the status quo--can require us to calculate disaggregated measures of a policy's effects to preserve the status quo. To see why, consider a simple case where a new policy increases the total income available to two people slightly, but reallocates income between them dramatically. Figure 5.3 illustrates this case. Start by assuming that transfers are costless. The status quo lies at A where the highest level of social welfare, W_A , is reached along the transfer possibilities curve $T_A(dy_2/dy_1 = -1 \text{ along } T_A)$. Policymakers clearly weight the two individuals equally here despite the fact that individual 2 receives more income. That is because the slope of W_A , $-W_1\lambda_1/W_2\lambda_2$ in the notation of Sec. IV, equals unity and hence $\alpha_1 = \alpha_2$. Now consider a policy that would move the individuals to B. If the individuals were to

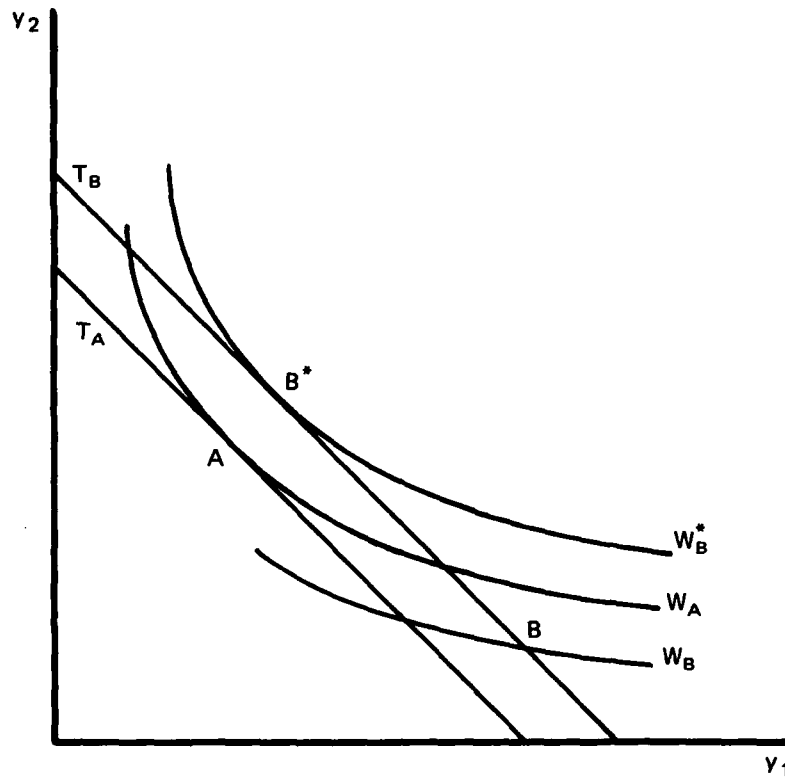


Fig. 5.3 – Comparison of two policies when income transfers are costless

remain at B, policymakers would no longer weight them equally. They clearly believe that individual 2 deserves more income and individual 1 less. That is because the slope of W_B is less than 1 in magnitude. But with costless transfers opening the possibility of B shifts the transfer possibilities curve out to T_B , thereby opening the possibility of point B^* where $\alpha_1 = \alpha_2$ again. If the transfer that moves these individuals from B to B^* were not available, policymakers would reject B because $W_A > W_B$. Only because the transfer to B^* is available do policymakers accept B--that is, B^* --and continue to weight individuals equally. In effect, the move from B to B^* is an integral part of the policy under consideration, despite the fact that equal weightings of the two individuals allow a decision to be made on B with aggregate data only.

Although explicit compensation need not be included as part of a specific policy package, the distributive effects of that policy must ultimately be offset by other policies to preserve an optimal

distribution of income. Ironically, Harberger, who makes perhaps the strongest plea for the use of aggregate measures as a professional convention, assumes a form of compensation that requires very accurate data on redistributive effects to be viable.

Historically, policymakers have not been inclined to use costless pure cash transfers of the type suggested in Fig. 5.3 to realize an optimal income distribution. Perhaps they are unavailable (cf. Becker, 1976). Consider briefly the implications of costly transfers for the analysis in Fig. 5.3. In Fig. 5.4, the status quo lies again at A, where now $\alpha_2 > \alpha_1$. That is because we assume A was reached by a transfer of income that transferred less than a dollar to individual 2 for every dollar given up by individual 1. Hence, the transfer did not go as far as it would have in Fig. 5.3, leaving individual 2 with relatively less than he would have had. Transfers back to individual 1 from A cannot reverse the effects of the original transfer, since less than a dollar could go to individual 1 for every dollar given up by individual 2. Hence A lies at a kink on the transfer possibility curve T_A .⁵ Now B becomes available. If transfers were costless, transfers

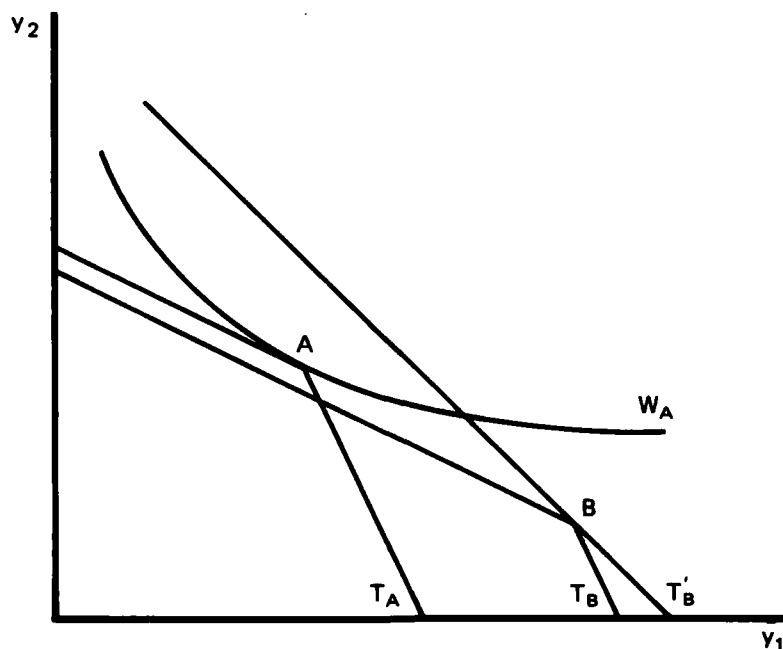


Fig. 5.4 – Comparison of two policies when income transfers are costly

⁵More generally, we can make transfers reversible along a concave T-function by making the act of transfer itself relatively costless, but

could be made along T'_B , with slope -1, allowing policymakers to reach a higher level of social welfare than W_A . But transfers are costly; along T_B , a level of social welfare as high as W_A cannot be reached. Hence, policymakers should reject B, despite the fact that an unweighted aggregate summation of individuals' incomes would suggest that B is better than A. When transfers are costly, a weighted measure is required. In this case, individual 2 will be weighted more heavily, indicating that he does not get as much income as he would have if transfers were costless.

As a general rule, of course, policymakers do not all agree that the current distribution of income is optimal in any sense. In essence, the examples above posit a social mechanism--perhaps the political process--that allocates income in a way that takes into account consistent social rankings of alternative allocations of income. That mechanism may reveal a set of weights over time that policymakers could use as a basis for consensus on the relative worth of groups of individuals.

To date, attempts to identify a stable set of such weights have not been successful.⁶ Until such weights can be identified, if ever, policy debates will continue to revolve around policy effects on many groups, whom decisionmakers value differently. Analyses of changes in consumer surplus will prove most useful to such debates if they eschew attempts to value groups relative to one another--that is the job of the political process--and address only effects on specific groups under discussion.

If analysis is to be useful, it must consider groups that are actually relevant to policymakers. The nature of such groups may be revealed in the course of debate, from previous debates on similar issues, or in the course of analysis when close attention to policy change reveals where its principal effects are likely to be focused. Groupings that economists are most comfortable with--quartiles or

putting limits on the ability to change incomes without changing incentives to produce or by associating allocative losses with transfers made through the price system. For example, see Camm (1976).

⁶For a discussion of this issue, see Steiner (1974).

deciles of personal income distribution or consumers and producers as generic classes--need not be the groupings that receive the most attention in debate. This is most important when policies are being tailored to effect a viable compromise. Tailoring raises issues about very specific groups whose identity often becomes known only as the process of compromise proceeds. This strongly suggests that the policy analyst who wishes to measure changes in consumer surplus must become familiar with the political process and, ultimately, become responsive to it. Only then can he be sure that his analysis of distributive effects will be useful.

One distribution issue of particular importance to the economist is the distinction between personal and functional distributions of income. As the analysis above reveals, the economist's welfare theory focuses heavily on the individual. The political process often does not. It very often sees the individual in his role as consumer of some product or owner of some factor. That is, the functional distribution of income is important to most policymakers. Hence, it will most often be useful to break down measures of surplus into those associated with consumers and owners of factors of various kinds. This presents no problems for any of our measures; but it does suggest that the use of the expenditure function to calculate changes in Hicksian measures has limitations. As Appendix A explains, the expenditure function focuses on the individual in all his activities. Only if it is broken down into components can it help us with questions about functional income redistribution.

The question of how small a group an analyst should examine poses a hard dilemma. On the one hand, the smaller the group examined, the more likely the analyst can help find solutions that promote consensus. On the other, with few exceptions, the accuracy of the analyst's measures falls as he moves to less aggregated data. This is true because data on more aggregated groups is easier to get, because analysis can be more complete when it must address a smaller number of--and hence more aggregated set of--groups, and because idiosyncrasies of individuals wash out as we consider more of them together. Presumably, a similar dilemma faces the policymaker, though he may be less aware of it. No general rules are available to suggest at what level analysis should concentrate. These arguments suggest, however, that whatever measure is

chosen, it should be able to proceed on the basis of relatively crude data on the behavior of individuals in a group. It should also give explicit attention to the uncertainties associated with any data used.

The choice of groups to analyze and the methods that specify our uncertainty about effects on any group are problems that transcend our choice among these three consumer surplus measures. All three can be adapted to deal with any type of group or level of aggregation. It is worth repeating at this point the problem of actual versus potential compensation. As noted earlier, this issue becomes more important as a group gets smaller because the probability that rents extracted from a specific group return to it in some other form falls as the group becomes smaller. Hence, Hicks and Harberger measures are hardest to justify for small groups unless actual compensation is contemplated and included in the policy package in question. By the same token, the quality of our data and information deteriorates as we examine smaller groups, making it harder to distinguish empirically among different measures.

CONCLUSION

As a general rule, we will not be able empirically to distinguish differences among Marshallian, Hicksian, and Harberger measures of changes in consumer surplus. They differ only because of income effects on demand and differences in contemplated compensation. So long as income effects are relatively small, the differences will be relatively small. They will be particularly small if we are uncertain about demand functions for the goods and services in question.

Nonetheless, occasions can arise when we must make a choice. Either income effects are large or aggregate measures after compensation are very sensitive to the choice. In this case, if we wish the measure chosen to be meaningful to policymakers, the choice hinges on which measure is most compatible with the form of compensation we expect to occur for the group in question. While none of the measures we have considered is perfect in this regard, the spirit underlying the Harberger measure appears to be the appropriate one. Harberger's explicit measure is inappropriate because it makes an arbitrary choice about compensation. We must find the demand function compatible with

the form of compensation we expect to occur. This suggests an important issue that has received no attention in the literature: How do we characterize the effects of compensation in kind--through goods and services instead of money--on the demand for or supply of goods being affected by a policy? Until this issue is resolved clearly, the "right" measure of consumer surplus remains to be found.

We emphasize here that this conclusion is not orthodox within the economics profession. Most economists prefer Hicksian measures because (a) they are integrable and (b) they yield measures that are compatible with concepts in traditional economics welfare theory. We support an alternative measure here because (a) although integrability is helpful, it is not essential, and (b) Hicksian measures are not generally compatible with notions important to policymakers in political debate. As a society, we choose to assign responsibility for making policy changes to political policymakers and not to economists. That suggests that the policymakers' needs should take priority over the economists' in our choice of the methods and tools of policy analysis. In the end, such a choice will prove fruitful both for the policymaker and for the economist.

Appendix A

THE RELATIONSHIP BETWEEN HICKSIAN CONSUMER SURPLUS AND THE EXPENDITURE FUNCTION

One key defense of Marshallian measures of consumer surplus, when comparing them with Hicksian measures, has always been that Marshallian measures are easy to construct using only observable market data on prices and quantities, whereas Hicksian measures are not. The availability of the expenditure function has made that argument harder to sustain. This appendix explains the expenditure function and illustrates its relationship to Hicksian and Marshallian demand concepts.

Consider a particular level of utility. Figure A.1 illustrates this by an indifference curve U^0 between two goods. Next, consider a budget line, $Y: y = \sum_i p_i x_i$ for fixed prices p_i . Holding these prices constant, increase income until the consumer is just able to attain U^0 . The corresponding income level, y^0 , is the "expenditure" associated with this level of utility and prices. A specific level of expenditure can be associated with any level of utility and any set of prices; this association is the consumer's "expenditure function":

$$E = E(U^*, p_1, \dots, p_n)$$

$$= \sum_i p_i x_i^*(p_1, \dots, p_n, y = E) \quad (A.1)$$

for $U = U(x_1, \dots, x_n)$. This function has a special property that makes it especially useful when measuring changes in consumer surplus. Hicksian demand functions are simple partial derivatives of the expenditure function. To see this, differentiate E with respect to p_j :

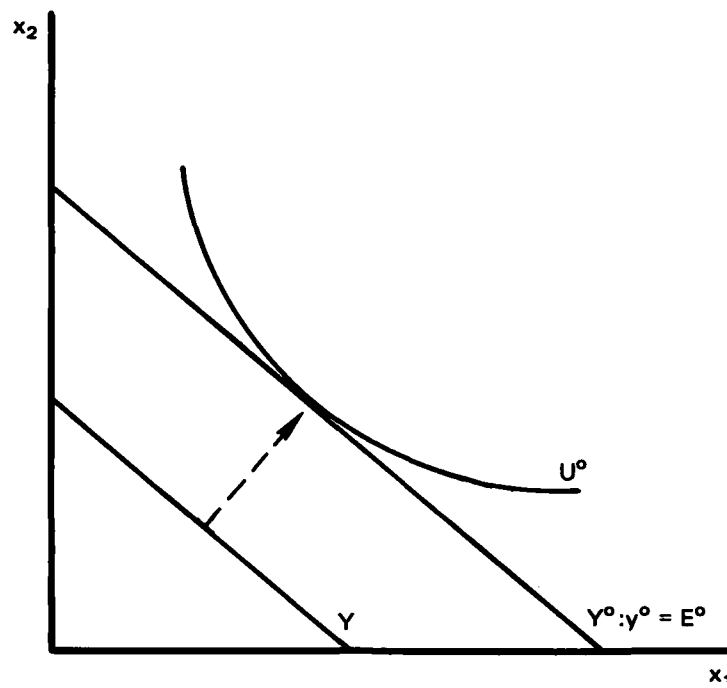


Fig. A.1 — Expenditure level required to achieve a fixed level of utility with fixed prices

$$\frac{\partial E}{\partial p_{1'}} = \sum_1 p_1 \frac{\partial x_1^*}{\partial p_j} + x_1^* \quad (\text{A.2})$$

where

$$\frac{\partial U^*}{\partial p_{1'}} = \sum_1 U_1 \frac{\partial x_1^*}{\partial p_{1'}} = \lambda \sum_1 p_1 \frac{\partial x_1^*}{\partial p_{1'}} = 0 \quad (\text{A.3})$$

Hence, substituting Eq. (A.3) into Eq. (A.2),

$$\frac{\partial E}{\partial p_i} = x_i^* \quad (A.4)$$

where x_i^* is the level of x_i consistent with U . This means that

$$\sum_i x_i^* dp_i = \sum_i \frac{\partial E}{\partial p_i} dp_i = dE \quad (A.5)$$

for $dU = 0$; the integrand of our Hicksian measures of consumer surplus is an exact differential of the expenditure function. Hence, when we measure changes in Hicksian consumer surplus, we are simply measuring changes in the expenditure function. If we can deduce the expenditure function, we can use it directly to make statements about consumer surplus instead of going through the potentially tedious process of summing areas to the left of consumption loci.

In fact, the expenditure function can contain a great deal of information about producer surplus as well. x_i^* in Eq. (A.4) is not restricted to be positive. When it is negative, it reflects a negative demand or, what is the same thing, a net supply of x_i . Such a Hicksian "demand" curve is shown in Fig. A.2. This view of the expenditure function is most appropriate in a pure exchange economy in which price changes are induced exogenously by policy changes outside the system. Production can be accommodated in this approach, but it becomes complex. Such an expenditure function cannot, without modification, accurately reflect changes in the welfare of individuals when policy changes drive wedges between consumer and producer prices for the same good. For the purposes of illustration, we will exclude the supply side from consideration altogether. This approach is consistent with that taken in Sec. II and the early part of Sec. III.

To deduce the expenditure function from observable data, we need to recognize two theoretical points. First, the expenditure function is the inverse of the "indirect utility function":

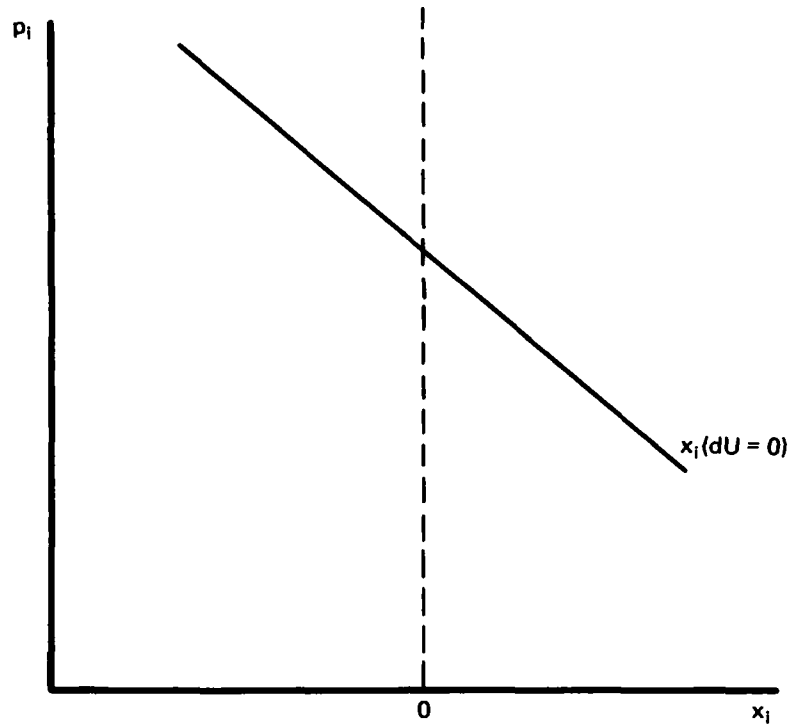


Fig. A.2 – Hicksian net demand

$$V = V(p_1, \dots, p_n, y)$$

$$= U[x_1(p_1, y), \dots, x_n(p_1, y)] \quad (A.6)$$

That is, for any set of prices and money income, there is only one highest level of utility that can be reached. In Fig. A.1, we could discover this level by choosing a budget line consistent with fixed income and prices and then varying utility until we discover the one utility level where the indifference curve is tangent to the budget line. The way we discover this utility level is just the reverse of how we discovered the expenditure level. For fixed prices, just one level of expenditure corresponds to each level of utility and vice versa. Hence, if we know prices and expenditure, we can find utility; if we

know prices and utility, we can find expenditure. We can find the expenditure function by inverting the indirect utility function.

Second, the indirect utility function is directly related to the Marshallian demand function. To see how, note first that along a Marshallian demand function,

$$\frac{\partial V}{\partial p_j} = \sum_i U_i \frac{\partial x_i}{\partial p_j} = \lambda \sum_i p_i \frac{\partial x_i}{\partial p_j} \quad (\text{A.7})$$

We know the consumer's income constraint, $y = \sum_i p_i x_i$, must bind so that

$$x_j + \sum_i p_i \frac{\partial x_i}{\partial p_j} = 0 \quad (\text{A.8})$$

Hence,

$$\frac{\partial V}{\partial p_j} = -\lambda x_j \quad (\text{A.9})$$

where x_j is Marshallian demand for the j th good. Then note that

$$\frac{\partial V}{\partial y} = \sum_i U_i \frac{\partial x_i}{\partial y} = \lambda \sum_i p_i \frac{\partial x_i}{\partial y} \quad (\text{A.10})$$

And from the consumer's budget constraint,

$$\sum_1 p_1 \frac{\partial x_1}{\partial y} = 1 \quad (\text{A.11})$$

or

$$\frac{\partial V}{\partial y} = \lambda \quad (\text{A.12})$$

Combining Eqs. (A.9) and (A.12),

$$-\frac{\partial V / \partial p_j}{\partial V / \partial y} = x_j \quad (\text{A.13})$$

The ratio of these two partials equals the Marshallian demand level.¹

Hence, we can move from an observable relationship in the Marshallian demand function through Eq. (A.13) to the indirect utility function. We can then invert the indirect utility function to obtain the expenditure function. And once the expenditure function is obtained, we can easily measure changes in Hicksian consumer surplus.

This can require some relatively advanced mathematics, but we can illustrate how this would work for a simple two-good case. Suppose we observe empirically that

$$x_1 = \frac{\alpha_1 y}{p_1} \quad x_2 = \frac{\alpha_2 y}{p_2} \quad (\text{A.14})$$

¹Equation (A.13) is known as Roy's identity. See Varian (1978, p. 93).

The indirect utility function

$$V = V(p_1, p_2, y) \quad (\text{A.15})$$

totally differentiated becomes

$$dV = \frac{\partial V}{\partial p_1} dp_1 + \frac{\partial V}{\partial p_2} dp_2 + \frac{\partial V}{\partial y} dy \quad (\text{A.16})$$

We can use values from the Marshallian demand function at a point on an indifference surface together with Eq. (A.16) to find:

$$\begin{aligned} \frac{dy}{dp_1} &= - \frac{\partial V / \partial p_1}{\partial V / \partial y} = x_1 = \frac{\alpha_1 y}{p_1} \\ \frac{dy}{dp_2} &= - \frac{\partial V / \partial p_2}{\partial V / \partial y} = x_2 = \frac{\alpha_2 y}{p_2} \\ \frac{dp_1}{dp_2} &= - \frac{\partial V / \partial p_2}{\partial V / \partial p_1} = - \frac{x_2}{x_1} = \frac{\alpha_2 p_1}{\alpha_1 p_2} \end{aligned} \quad (\text{A.17})$$

or

$$\frac{dy}{y} = \alpha_1 \frac{dp_1}{p_1} \quad \frac{dy}{y} = \alpha_2 \frac{dp_2}{p_2} \quad - \alpha_1 \frac{dp_1}{p_1} = \alpha_2 \frac{dp_2}{p_2} \quad (\text{A.18})$$

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CONSUMER SURPLUS DEMAND FUNCTIONS AND POLICY ANALYSIS
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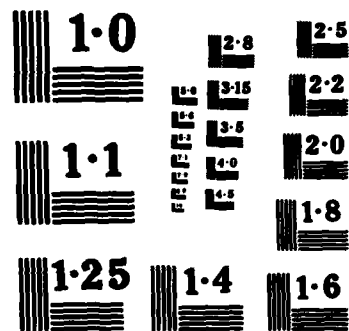
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Solving these differential equations yields

$$\begin{aligned}\ln y - \alpha_1 \ln p_1 &= k_1(p_2, \bar{V}) \\ \ln y - \alpha_2 \ln p_2 &= k_2(p_1, \bar{V}) \\ \alpha_1 \ln p_1 + \alpha_2 \ln p_2 &= k_{12}(y, \bar{V})\end{aligned}\tag{A.19}$$

These are all consistent with an indirect utility function

$$g(\bar{V}) = \ln y - \alpha_1 \ln p_1 - \alpha_2 \ln p_2\tag{A.20}$$

where $g(\bar{V})$ is a constant that represents the level of utility on the relevant indifference surface. Inverting Eq. (A.20) yields an expenditure function

$$\ln E = g(\bar{V}) + \alpha_1 \ln p_1 + \alpha_2 \ln p_2\tag{A.21}$$

which we can use to calculate Hicksian measures of consumer surplus.

We can verify that Eq. (A.20) is correct by noting that Eqs. (A.14) are Marshallian demand functions derived from a Cobb-Douglas utility function:

$$U = x_1^{\alpha_1} x_2^{\alpha_2}\tag{A.22}$$

for $\alpha_1 + \alpha_2 = 1$. Utility maximization yields

$$\lambda p_1 = U_1 = \frac{\alpha_1 U}{x_1} \quad (\text{A.23})$$

so that

$$U = \alpha_1^{\alpha_1} \alpha_2^{\alpha_2} p_1^{-\alpha_1} p_2^{-\alpha_2} (U/\lambda) \quad (\text{A.24})$$

from the income constraint and Eq. (A.23)

$$y = (\alpha_1 + \alpha_2) \left(\frac{U}{\lambda} \right) = \left(\frac{U}{\lambda} \right) \quad (\text{A.25})$$

Hence, Eq. (A.24) becomes

$$x_1 = \frac{\alpha_1}{p_1} \left(\frac{U}{\lambda} \right) = \frac{\alpha_1 y}{p_1} \quad (\text{A.26})$$

which are the Marshallian demand functions in Eqs. (A.14). Further, Eq. (A.24) becomes

$$U \left(\alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2} \right) = p_1^{\alpha_1} p_2^{\alpha_2} y$$

which is equivalent to the indirect utility function in Eq. (A.20). The expenditure function follows directly:

$$E = \left(\alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2} \right) p_1^{\alpha_1} p_2^{\alpha_2} U \quad (\text{A.27})$$

This is equivalent to Eq. (A.21). Substituting Eq. (A.27) into Eq. (A.26) properly compensates an individual for price changes to yield the Hicksian demand functions:

$$\begin{aligned}
 x_1 &= \frac{\alpha_1 \left(\frac{\alpha_1}{\alpha_2} \right)^{\alpha_2} p_1^{\alpha_1} p_2^{\alpha_2} U}{p_1} \\
 &= \left(\frac{\alpha_1}{\alpha_2} \right)^{\alpha_2} \left(\frac{p_2}{p_1} \right)^{\alpha_2} U \\
 x_2 &= \left(\frac{\alpha_2}{\alpha_1} \right)^{\alpha_1} \left(\frac{p_1}{p_2} \right)^{\alpha_1} U
 \end{aligned}
 \tag{A.28}$$

It is easy to verify that these are precisely the partial derivatives of the expenditure function in either Eq. (A.21) or Eq. (A.27).

Similar procedures allow us to recover the expenditure function underlying other empirical Marshallian demand functions. The mathematics simply becomes more difficult. For more detailed information on this process, see Diamond and McFadden (1974) and Hausman (1981).

Appendix B

CONSTRUCTION OF REACTION FUNCTIONS

Only in the simplest case where a good has no substitutes or the supply of all substitutes for a good is infinitely elastic *and* policy changes affect only the price of this one good can we use the area to the left of a demand curve to get measures of consumer surplus. Other cases benefit from the use of reaction functions to generate consumption loci. This appendix first explains how to generate consumption loci and reaction functions for two goods and then considers the n-good case.

TWO GOODS

Suppose we can identify simple linear demand and supply curves for two substitutes, the first of which faces a tax:

$$\tilde{D}_1 = a_{11}\tilde{p}_1 + a_{12}\tilde{p}_2$$

$$\tilde{D}_2 = a_{21}\tilde{p}_1 + a_{22}\tilde{p}_2 \quad (B.1)$$

$$\tilde{S}_1 = b_1(\tilde{p}_1 - t_1)$$

$$\tilde{S}_2 = b_2p_2$$

where $\tilde{D}_1 \equiv D_1 - D_1^0$, $\tilde{p}_1 \equiv p_1 - p_1^0$, and so on for a null superscript that denotes values when $t_1 = 0$. How would changes in a tax on the

first good affect its consumption? For any value of t_1 , the market will clear:

$$b_1(\tilde{p}_1 - t_1) = a_{11}\tilde{p}_1 + a_{12}\tilde{p}_2$$

$$b_2\tilde{p}_2 = a_{21}\tilde{p}_1 + a_{22}\tilde{p}_2$$

$$\begin{bmatrix} (a_{11} - b_1) & a_{12} \\ a_{21} & (a_{22} - b_2) \end{bmatrix} \begin{bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \end{bmatrix} = \begin{bmatrix} -b_1 t_1 \\ 0 \end{bmatrix} \quad (\text{B.2})$$

Solving this linear system yields

$$\Delta\tilde{p}_1 = -b_1(a_{22} - b_2)t_1$$

$$\Delta\tilde{p}_2 = b_1 a_{21} t_1 \quad (\text{B.3})$$

$$\text{for } \Delta = (a_{11} - b_1)(a_{22} - b_2) - a_{12}a_{21}$$

Substituting Eq. (B.3) into Eq. (B.1) yields

$$\Delta\tilde{D}_1 = -a_{11}a_{22}b_1 + (a_{11}b_2 + a_{12}a_{21})b_1 t_1 \quad (\text{B.4})$$

Solving for t_1 in Eq. (B.3) and substituting into Eq. (B.4) yields the reaction function for the taxed good:

$$\tilde{d}_1 = - \frac{a_{11}a_{22}b_1}{\Delta} - \frac{(a_{11}b_2 + a_{12}a_{21})}{a_{22} - b_2} t_1 \quad (B.5)$$

Note that when b_2 approaches infinity, causing \tilde{p}_2 to approach zero, substitution effects are no longer important and Eq. (B.5) approaches the appropriate expression in Eq. (B.1).

n GOODS

Define

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{nn} \end{bmatrix}$$

$$\tilde{p} = \begin{bmatrix} \tilde{p}_1 \\ \vdots \\ \tilde{p}_n \end{bmatrix} \quad \underline{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_n \end{bmatrix} \quad \tilde{d} = \begin{bmatrix} \tilde{d}_1 \\ \vdots \\ \tilde{d}_n \end{bmatrix} \quad \tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \vdots \\ \tilde{s}_n \end{bmatrix}$$

Then we can restate Eq. (B.1) for n goods as

$$\tilde{d} = A\tilde{p} \quad \tilde{s} = B(\tilde{p} - \underline{t}) \quad (B.6)$$

Proceeding as we did for two goods yields

$$\tilde{\underline{p}} = (B - A)^{-1} B t$$

$$\tilde{\underline{D}} = A(B - A)^{-1} B t \quad (B.7)$$

Prices and quantities are again parameterized in terms of t . We can use Eq. (B.7) to derive expressions like Eqs. (B.3) and (B.4) for any *single* good, solve for t_i in the price equation, and derive an appropriate reaction function like that in Eq. (B.5).

Equation (B.7) provides another important piece of information. It fully defines demand levels for any set of taxes:

$$D_i = D_i(t_1, \dots, t_n) \quad (B.8)$$

As should be clear from the text, we can use

$$\int \sum_i D_i dt_i = \int \sum_i X_i dp_i - \int \sum_i X_i dc_i \quad (B.9)$$

as a measure of the sum of changes in consumer and producer surpluses. Such an expression will be integrable if and only if

$$\frac{\partial D_i}{\partial t_{i'}} = \frac{\partial D_{i'}}{\partial t_i} \quad \text{for all } i, i' \quad (B.10)$$

that is, if $A(B - A)^{-1}B$ is symmetric. Note that the inverse of this expression is $B^{-1}(B - A)A^{-1} = A^{-1} - B^{-1}$. If A and B are symmetric, then $A^{-1} - B^{-1}$ is symmetric, and $A(B - A)^{-1}B$ is symmetric. A is symmetric if $\partial x_i / \partial p_i' = \partial x_{i'} / \partial p_i$, our integrability condition without reaction curves. The symmetry of B depends on the same sort of condition. It will

obviously be fulfilled if production sectors are independent of one another and B is diagonal. In this case, the symmetry of A is necessary and sufficient for Eq. (B.10) to hold. The key here is that if the basic conditions that assure integrability for individuals hold when reaction curves are not necessary, they also hold along reaction curves.

Although we have set Eq. (B.6) up as a linear system over discrete ranges of quantity and price, it could equally well represent a set of total differentials. Equation (B.7) would then spell out comparative statics that could be used to characterize incremental movements along reaction curves. So long as $A(B - A)^{-1}B$ is symmetric, Eq. (B.10) is satisfied and Eq. (B.9) is integrable regardless of the specific shape of the demand, supply, or reaction curves. Hence, the results shown here, properly applied, easily generalize beyond the linear case.

Appendix C

COMPENSATED PRICE EFFECTS FOR A BAILEY DEMAND FUNCTION

Harberger measures depend on Bailey demand functions. To understand the integrability of measures based on these functions, we need to understand the form that cross price effects take for such functions. This appendix examines that question.

Given a Marshallian demand function

$$x_1 = f^1(p_1, \dots, p_n, y) \quad (C.1)$$

we define the Bailey cross price effect

$$\left. \frac{\partial x_1}{\partial p_j} \right|_B = \frac{\partial f^1}{\partial p_j} + \frac{\partial f^1}{\partial y} \frac{\partial y}{\partial p_j} \Big|_B \quad (C.2)$$

using the relationship between income and the j th price, $\partial y / \partial p_j$. To identify $(\partial y / \partial p_j)_B$, recall that a Bailey demand function is defined by imposing a restriction on the set of goods that can be consumed. In Sec. II, we define demand by restricting the individual to his initial budget line, $\sum_i c_i x_i = y$, where the c_i are exogenously set, net-of-tax prices. Fully differentiating, this expression yields

$$\sum_i c_i dx_i = 0 \quad (C.3)$$

which essentially defines the set of consumption changes allowed along a Bailey demand function.

What adjustments in income are required to effect Eq. (C.3)?
Totally differentiate the individual's budget constraint:

$$dy = \sum_1 (c_1 + t_1) dx_1 + \sum_1 x_1 dp_1 \quad (C.4)$$

For Eq. (C.3) to hold, income must change so that Eq. (C.4) holds when Eq. (C.3) is substituted in:

$$dy = \sum_1 t_1 dx_1 + \sum_1 x_1 dp_1 \quad (C.5)$$

x_1 is of course a function of price and income. For

$$x_1 = f^1(p_1, \dots, p_n, y)$$

we have the following result for a change in p_j :

$$dx_1 = \left(\frac{\partial f^1}{\partial p_j} + \frac{\partial f^1}{\partial y} \frac{\partial y}{\partial p_j} \Big|_B \right) dp_j \quad (C.6)$$

To define $(\partial y / \partial p_j)_B$ substitute Eq. (C.6) into Eq. (C.5):

$$dy = \left(\sum_1 t_1 \frac{\partial f^1}{\partial p_j} + x_j \right) dp_j + \sum_1 t_1 \frac{\partial f^1}{\partial y} \frac{\partial y}{\partial p_j} \Big|_B dp_j$$

or, when only y and p_j change together,

$$\left. \frac{\partial y}{\partial p_j} \right|_B = \frac{\sum_i t_i \frac{\partial f^i}{\partial p_j} + x_j}{1 - \sum_i t_i \frac{\partial f^i}{\partial y}} \quad (C.7)$$

From the individual's point of view, we know that

$$\frac{\partial y}{\partial p_j} = \frac{\partial}{\partial p_j} \sum_i p_i x_i = \sum_i c_i \frac{\partial x_i}{\partial p_j} + \sum_i t_i \frac{\partial x_i}{\partial p_j} + x_j = 0$$

and

$$\frac{\partial y}{\partial y} = \frac{\partial}{\partial y} \sum_i p_i x_i = \sum_i c_i \frac{\partial x_i}{\partial y} + \sum_i t_i \frac{\partial x_i}{\partial y} = 1$$

Substituting from Eq. (C.8) into Eq. (C.7) yields

$$\left. \frac{\partial y}{\partial p_j} \right|_B = - \frac{\sum_i c_i \frac{\partial f^i}{\partial p_j}}{\sum_i c_i \frac{\partial f^i}{\partial y}} \quad (C.9)$$

In the terms of Sec. II, then, Eq. (C.2) becomes

$$\left. \frac{\partial x_1}{\partial p_j} \right|_B = \left. \frac{\partial x_1}{\partial p_j} \right|_M - \left[\frac{\sum_1 c_1 \frac{\partial f^1}{\partial p_j}}{\sum_1 c_1 \frac{\partial f^1}{\partial y}} \right] \frac{\partial x_1}{\partial y} \quad (C.10)$$

Using Eq. (2.21), we can see that the relationship between the Bailey and Hicks cross price effects is

$$\left. \frac{\partial x_1}{\partial p_j} \right|_B = \left. \frac{\partial x_1}{\partial p_j} \right|_H - \left[x_j + \frac{\sum_1 c_1 \frac{\partial f^1}{\partial p_j}}{\sum_1 c_1 \frac{\partial f^1}{\partial y}} \right] \frac{\partial x_1}{\partial y} \quad (C.11)$$

For the Bailey demand function to be integrable, we need $(\partial x_1 / \partial p_j)_B = (\partial x_j / \partial p_1)_B$. This holds for the Hicks function. This will be true for the Bailey function only if the second term in Eq. (C.11) obeys a similar cross condition. Such a condition is no longer met when income elasticities are equal. Hence a Bailey demand function may not be integrable even when both Hicksian and Marshallian functions are. The complexity of the expression in Eq. (C.11) suggests that intuitively appealing conditions under which a Bailey demand function is integrable would be hard to define.

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